TENSEGRITY SHAPE CONTROL

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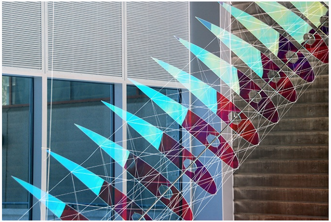
Vehicle Systems & Control Laboratory

INTRODUCTION

Tensegrity Structures:

- A configuration of axially-loaded members (sticks and strings) stabilized by string tension
- Minimum-mass optimization
- Deployable / reconfigurable applications



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Shape Control Approach:

- *Premise*: Equilibrium configurations change with string tensions
- Describe dynamics in terms of node positions
- Specify shape objective
- Drive error between current and desired node positions to zero by adjusting tensions

Potential Shape Control Applications:

- Morphing Airfoil
- Robotic Arm
- Formation Control

TENSEGRITY DEFINITIONS AND DYNAMICS

Member Matrices:

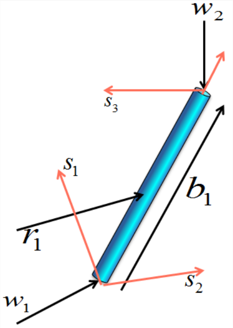
$$B = [b_1 \dots b_\beta], \quad S = [s_1 \dots s_\alpha], \quad R = [r_1 \dots r_\beta]$$
$$N = [n_1 \dots n_{2\beta}], \quad W = [w_1 \dots w_{2\beta}]$$

Connectivity Matrices:

$$\begin{bmatrix} B & S & R \end{bmatrix} = N \begin{bmatrix} C_b^T & C_s^T & C_r^T \end{bmatrix}$$
$$C_b = \begin{bmatrix} -I & I \end{bmatrix}, \quad C_r = \tfrac{1}{2} \begin{bmatrix} I & I \end{bmatrix}$$

Internal Forces:

$$\gamma_i = \frac{\text{force in string } s_i}{\|s_i\|}, \quad \lambda_i = \frac{\text{force in bar } b_i}{\|b_i\|}$$
$$m_i = \text{mass of bar } b_i$$



Vector Nomenclature

Class 1 Tensegrity Dynamics:

$$\ddot{N}M + NK(\gamma) = W$$
$$M \equiv \frac{1}{12}C_b^T\hat{m}C_b + C_r^T\hat{m}C_r$$
$$K \equiv C_s^T\hat{\gamma}C_s + C_b^T\hat{\lambda}C_b$$

where

$$\hat{\lambda} \equiv \lfloor \dot{B}^T\dot{B} \rfloor \hat{l}^{-2}\hat{m}\frac{1}{12} + \lfloor B^T F C_b^T \rfloor \hat{l}^{-2}\frac{1}{2}$$
$$F(\gamma) = W - S\hat{\gamma}C_s$$

CONTROL LAW

Need method for specifying the **desired final shape** of the given tensegrity structure.

L: $j \times 3$ matrix, specifies the **"axes of interest"**

R: $n \times h$ matrix, specifies **"nodes of interest"**

LNR extracts the current values of the "node coordinates of interest"

$$Y_c = LNR \quad \in \quad \Re^{j \times h} \tag{1}$$

Y describes **error** between current (Y_c) and desired ($\bar{Y}$) "node coordinates of interest" values:

$$\begin{aligned} Y &= Y_c - \bar{Y} && (2) \\ &= LNR - \bar{Y} && (3) \end{aligned}$$

Desired error dynamics:

$$\ddot{Y} + \Psi\dot{Y} + \Omega Y = 0 \tag{4}$$

Express error dynamics in terms of N:

$$Y = LNR - \bar{Y} \tag{5}$$

$$\dot{Y} = L\dot{N}R \tag{6}$$

$$\ddot{Y} = L\ddot{N}R \tag{7}$$

$$\therefore \boxed{L\ddot{N}R + \Psi L\dot{N}R + \Omega(LNR - \bar{Y}) = 0} \tag{8}$$

Recall full system dynamics in terms of N:

$$\ddot{N}M + NK(\gamma) = W$$

$$M \equiv \frac{1}{12} C_b^T \hat{m} C_b + C_r^T \hat{m} C_r$$

$$K \equiv C_s^T \hat{\gamma} C_s - C_b^T \hat{\lambda} C_b$$

$$\hat{\lambda} \equiv \lfloor \dot{B}^T \dot{B} \rfloor \hat{l}^{-2} \hat{m} \frac{1}{12} + \lfloor B^T F C_b^T \rfloor \hat{l}^{-2} \frac{1}{2}$$

Re-express λ for ith bar member:

$$\lambda = \Lambda\gamma - \tau$$

$$\tau_i = \frac{m_i}{12 l_i^2} C_b \mathbf{e}_i^T \dot{N}^T \dot{N} \mathbf{e}_i C_b + \frac{1}{2 l_i^2} (\mathbf{e}_i^T C_b N^T) W C_b^T \mathbf{e}_i$$

$$\Lambda_i = \frac{1}{2 l_i^2} \mathbf{e}_i^T C_b N^T N C_s^T C_s C_b^T \mathbf{e}_i$$

Substitute derived system dynamics:

$$\ddot{N} = (W - NK)M^{-1}$$

$$\downarrow$$

$$L\ddot{N}R + \Psi L\dot{N}R + \Omega(LNR - \bar{Y}) = 0$$

$$\boxed{LNKM^{-1}R} = LWM^{-1}R + \Psi L\dot{N}R + \Omega LNR - \Omega\bar{Y}$$

Substitute for K in first term and rearrange. γ appears linearly:

$$LNKM^{-1}R\mathbf{e}_i = LN\left[C_s^T(C_sM^{-1}R\mathbf{e}_i)^\wedge\gamma - C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\lambda\right]$$

Substitute $\lambda = \Lambda\gamma - \tau$ and combine γ terms:

$$LNKM^{-1}R\mathbf{e}_i =$$

$$LN\left[\left[C_s^T(C_sM^{-1}R\mathbf{e}_i)^\wedge + C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\Lambda\right]\gamma + C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\tau\right]$$

Full expression for desired error dynamics:

$$\big[LWM^{-1}R + \Psi L\dot{N}R + \Omega(LNR - \bar{Y})\big]\mathbf{e}_i$$
$$= LN\Big[\big[C_s^T(C_sM^{-1}R\mathbf{e}_i)^\wedge + C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\Lambda\big]\gamma + C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\tau\Big]$$

Isolate γ term:

$$\big[LWM^{-1}R + \Psi L\dot{N}R + \Omega(LNR - \bar{Y})\big]\mathbf{e}_i - LNC_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\tau$$
$$= LN\big[C_s^T(C_sM^{-1}R\mathbf{e}_i)^\wedge + C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\Lambda\big]\gamma \quad (9)$$

This is effectively of the form: $\mu = \Gamma\gamma$

Define μ and Γ appropriately, enforcing **non-negative** tensions, and solve for γ:

$$\begin{aligned} \mu =& \Gamma\gamma, \qquad \gamma \geq 0 \\ \mu_i =& \big[LWM^{-1}R + \Psi L\dot{N}R + \Omega(LNR - \bar{Y})\big]\mathbf{e}_i \\ & - LNC_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\tau \\ \Gamma_i =& LN\big[C_s^T(C_sM^{-1}R\mathbf{e}_i)^\wedge + C_b^T(C_bM^{-1}R\mathbf{e}_i)^\wedge\Lambda\big] \end{aligned}$$

where

$$\mu = \begin{bmatrix}\mu_1 & \mu_2 & \dots & \mu_n\end{bmatrix}^T$$
$$\Gamma = \begin{bmatrix}\Gamma_1 & \Gamma_2 & \dots & \Gamma_n\end{bmatrix}^T$$

SIMULATION RESULTS

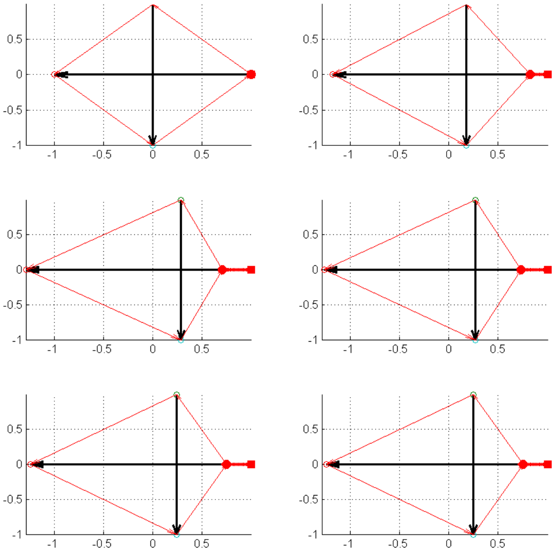
2D Cross Case 1:

Node 1 to **x = 0.75**

$$L = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\bar{Y} = 0.75$$



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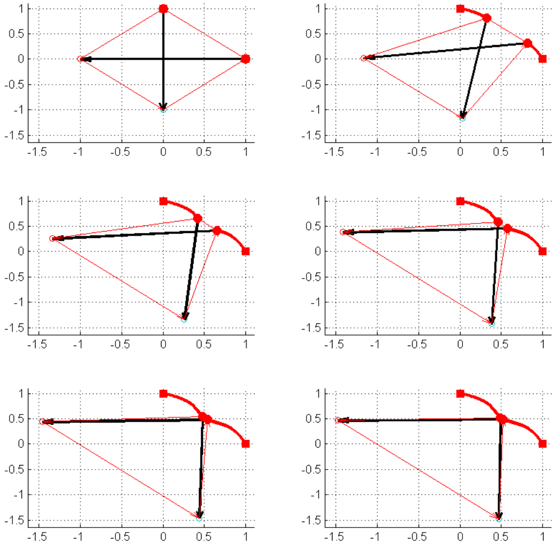
2D Cross Case 2:

Nodes 1 and 2 to (0.5,0.5)

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{Y} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$



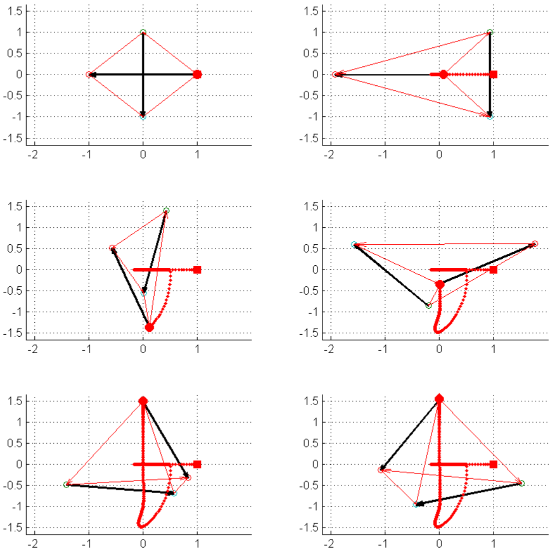
2D Cross Case 3:

Node 1 to $\mathbf{x} = \mathbf{0}$:

$$L = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

$$R = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$$

$$\bar{Y} = 0$$



2D Cross Case Study:

Collapse cross to $\mathbf{x} = \mathbf{0}$

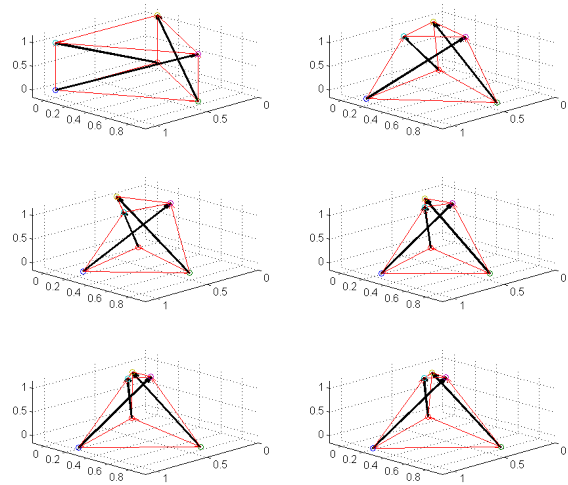
(See animations)

3D Prism Case 1:
Transform to pyramid

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R = I_6$$

$$\bar{Y} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.87 \\ 0 & 0 \\ 0.5 & 0.29 \\ 0.5 & 0.29 \\ 0.5 & 0.29 \end{bmatrix}^T$$



simulation results

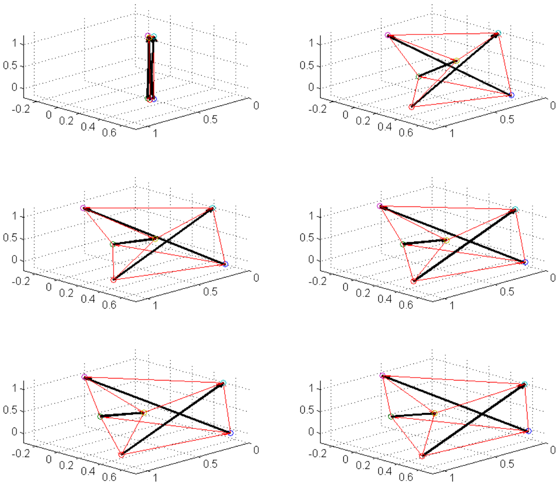
3D Prism Case 2:

Deployability

$$L = I_3$$

$$R = I_6$$

$$\bar{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.87 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0.5 & 0.87 & 1 \\ 0 & 0 & 1 \end{bmatrix}^T$$



CONCLUSIONS AND FUTURE WORK

Conclusions:

- Control law is functioning correctly
- Shape control method scaled without modification
- Solutions can be found with non-negative constraint on tensions

Future Work:

- Assess scalability with more complex structures
- Apply to structure *design* task
- Detect structure self-interference
- Include external forces and disturbances
- Demonstrate on example applications

Questions?