Global Tracking Control Structures for Nonlinear Singularly Perturbed Aircraft Systems

Anshu Siddarth and John Valasek

Department of Aerospace Engineering

Texas A&M University

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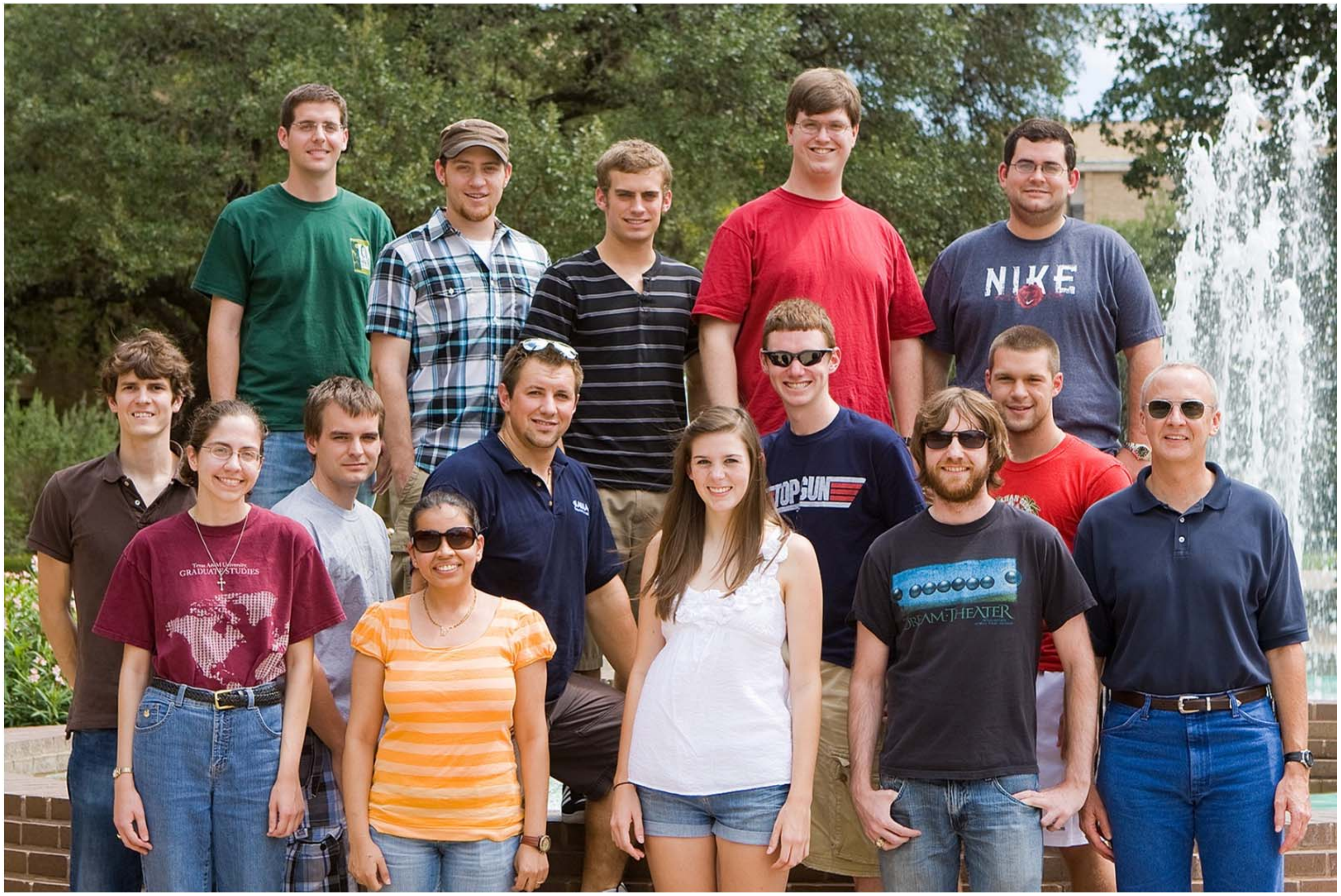
Student Research Team
2010 - 2011

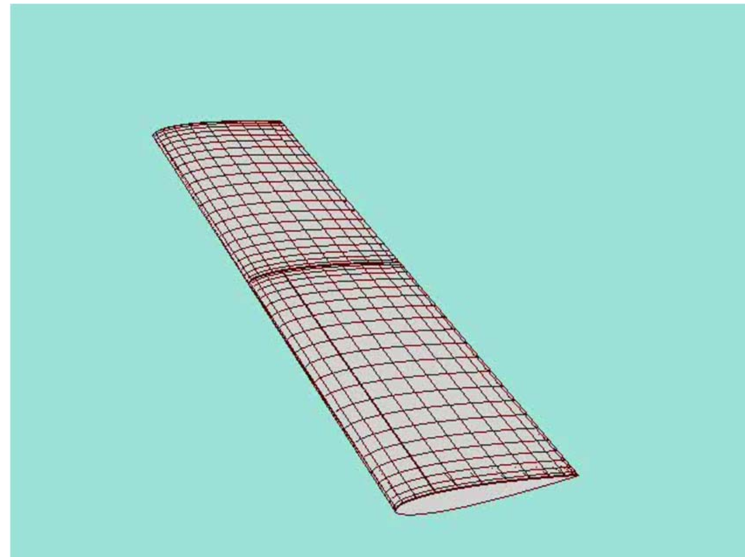


Outline

- Problem Statement
  - Simultaneous Slow and Fast State Tracking
  - Literature Review
- Geometric Singular Perturbation Theory
- Mathematical Development & Stability Analysis
- Numerical Simulations
  - Nonlinear Six Degree-of-Freedom Simulation of an F/A-18A Hornet
- Conclusions & Future Work

Motivations

10+ Independent Morphing DOF



$$\dot{\sigma} = f(\sigma, \omega)$$
$$\dot{\omega} = g(\sigma, \omega) + h(\sigma, \omega, p)u$$

Affine in Control

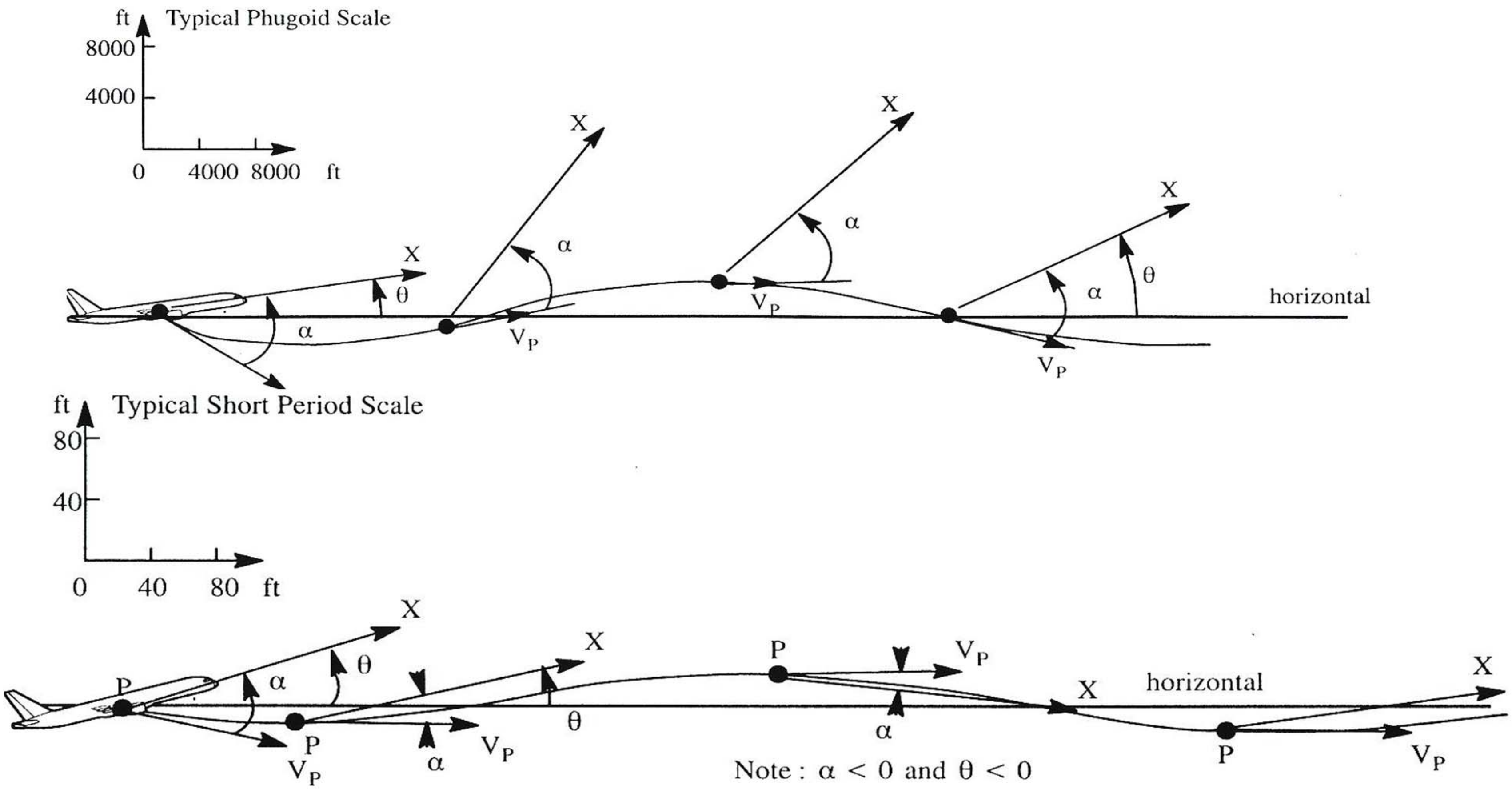
→

$$\dot{\sigma} = f(\sigma, \omega)$$
$$\dot{\omega} = g(\sigma, \omega, u) + h(\sigma, \omega, p, u)$$

Non-Affine in Control

Two Time Scale

Two-Time Scale Systems



Research Objective

- Nonlinear tracking control structures for:
  - **Two-Time Scale Systems/ Singularly Perturbed Systems**

    Examples: mechanical oscillators, airplanes, flexible robot link manipulators, …
  - **Mathematical form:**

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{z}) + \mathbf{g}(\mathbf{x},\mathbf{z})\mathbf{u}$$

$$\epsilon\dot{\mathbf{z}} = \mathbf{l}(\mathbf{x},\mathbf{z}) + \mathbf{k}(\mathbf{x},\mathbf{z})\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \mathbf{z} \end{bmatrix}$$

$\mathbf{x}$ is the vector of slow variables,

$\mathbf{z}$ is the vector of fast variables,

ϵ is a small positive parameter that captures the time scale property (unknown)

y is the vector of outputs.

Literature Survey

- Nonlinear Two-Time Scale System Analysis **Mease et. al**, (2003)
- Tracking of **slow variables** using the composite of:
  - Tracking controller for the slow subsystem,
  - Stabilizing controller for the fast subsystem to restrict fast states onto a manifold
- Global tracking results guaranteed only if the manifold is **unique**
  - Nonlinear in slow states and linear in fast states, **Li** (2009)
  - Assume unique manifold, **Grujic** (1988), **Choi** (2005)
- For general nonlinear systems local stability proven:
  - Assume the fast states as control variables for slow system, **Menon** (1987)
  - Approximate manifold, **Siddarth and Valasek** (*JGCD* May-Jun 2011)
- Simultaneous slow and fast tracking posed as optimal control problem, **Arstein** (1997)

Challenges: Nonlinear Singularly-Perturbed Model

- **Key Issues:**
  - Numerically stiff equations
  - Fast states are restricted to lie on a manifold
  - Global results valid only if fast variables lie on a **unique** manifold
- **Approach:**
  - Model-reduction via Geometric Singular Perturbation Theory
  - Use coordinate transformation and enforce the manifold to be exactly the fast state reference
  - Composite control design
- **Benefits:**
  - No assumptions on the class of nonlinear systems considered
  - Does not require computation of the manifold
  - Global asymptotic tracking
  - No knowledge of the singular perturbation parameter required

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Geometric Singular Perturbation Theory

(Fenichel, 1979)

Derivatives wrt slow time scale t

$$\dot{x}_1 = -x_1 - z$$
$$\dot{x}_2 = -x_2 - z$$
$$\epsilon \dot{z} = -z$$

$$\tau = \frac{t - t_0}{\epsilon}$$

$$x_1{'} = -\epsilon x_1 - \epsilon z$$
$$x_2{'} = -\epsilon x_2 - \epsilon z$$
$$z{'} = -z$$

Derivatives wrt fast time-scale τ

- Develop reduced-order models. Substitute $\epsilon = 0$

$$z = 0$$
$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_2$$

Slow Subsystem

$$x_1{'} = 0$$
$$x_2{'} = 0$$
$$z{'} = -z$$

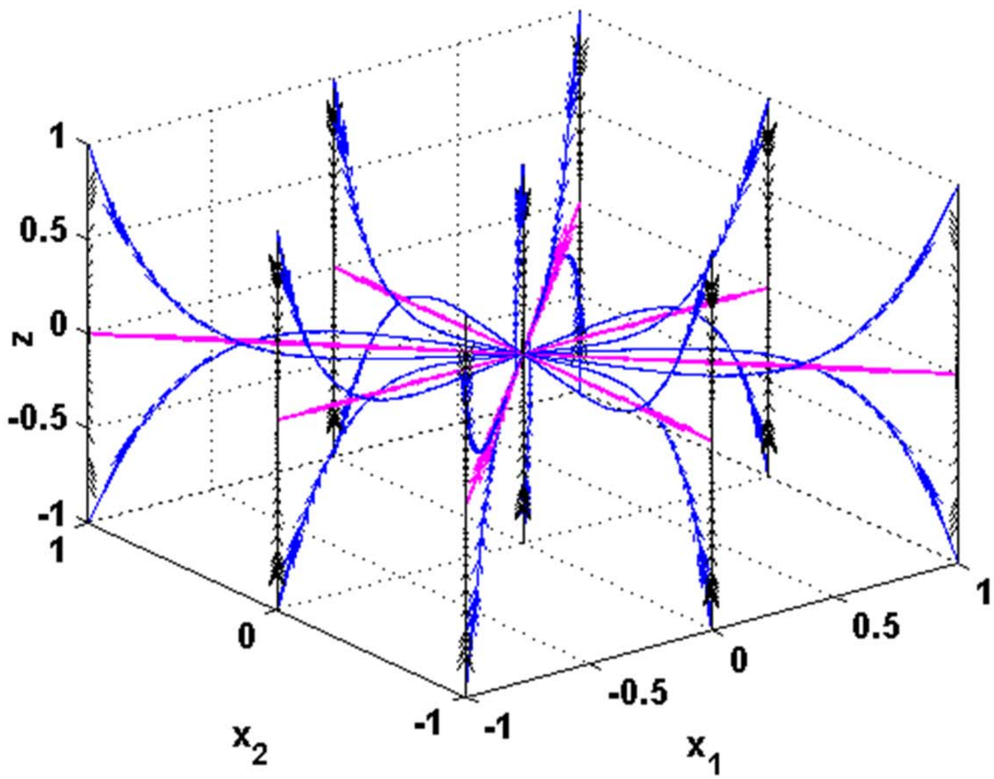
Fast Subsystem

Geometric Singular Perturbation Theory

- Reduced-order models approximate the behaviour of the complete system

Complete System

$$\dot{x}_1 = -x_1 - z$$
$$\dot{x}_2 = -x_2 - z$$
$$\epsilon \dot{z} = -z$$



Slow Subsystem

$$z = 0$$
$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_2$$

Complete
Slow Subsystem
Fast Subsystem

Fast Subsystem

$$x_1' = 0$$
$$x_2' = 0$$
$$z' = -z$$

Geometric Singular Perturbation Theory

Insights From The Geometric Approach

- Complete system (in blue) approximated by dynamics of slow subsystem (in pink) and fast subsystem (in black).
  - if complete system locally flattened it falls onto dynamics of the slow sub-system.

- Manifold is described by solution of the algebraic equation when substituting epsilon = 0 in complete system.

- Dynamics on manifold governed by differential equations of slow subsystem which is exactly the dynamics of the slow variables.

- Fast dynamics need to lie on the manifold.

Geometric Singular Perturbation Theory

Insights From The Geometric Approach

Key Idea: Tracking of both slow and fast states is achieved if and only if the manifold is exactly the reference trajectory of the fast states.

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Mathematical Formulation of the Control Law

- Complete Model:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{z}) + \mathbf{g}(\mathbf{x},\mathbf{z})\mathbf{u}$$
$$\epsilon\dot{\mathbf{z}} = \mathbf{l}(\mathbf{x},\mathbf{z}) + \mathbf{k}(\mathbf{x},\mathbf{z})\mathbf{u}$$
$$\mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \mathbf{z} \end{bmatrix}$$

- Tracking errors:

$$\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{x}_r(t)$$
$$\xi(t) = \mathbf{z}(t) - \mathbf{z}_r(t)$$

- Transform the equilibrium to origin:

$$\dot{\mathbf{e}} = \mathbf{f}(\mathbf{x},\mathbf{z}) + \mathbf{g}(\mathbf{x},\mathbf{z})\mathbf{u} - \dot{\mathbf{x}}_r \triangleq \mathbf{F}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r,\dot{\mathbf{x}}_r) + \mathbf{G}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r)\mathbf{u}$$
$$\epsilon\dot{\boldsymbol{\xi}} = \mathbf{l}(\mathbf{x},\mathbf{z}) + \mathbf{k}(\mathbf{x},\mathbf{z})\mathbf{u} - \epsilon\dot{\mathbf{z}}_r \triangleq \mathbf{L}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r,\epsilon\dot{\mathbf{z}}_r) + \mathbf{K}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r)\mathbf{u}$$

Step 1. Obtain Reduced-Order Models

$$\dot{\mathbf{e}} = \mathbf{F}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \dot{\mathbf{x}}_r) + \mathbf{G}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)\mathbf{u}$$

$$\epsilon\dot{\xi} = \mathbf{L}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \epsilon\dot{\mathbf{z}}_r) + \mathbf{K}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)\mathbf{u}$$

- Reduced Slow Subsystem

$$\dot{\mathbf{e}} = \mathbf{F}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \dot{\mathbf{x}}_r) + \mathbf{G}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)\mathbf{u}_0$$

$$\mathbf{0} = \mathbf{L}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \mathbf{0}) + \mathbf{K}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)\mathbf{u}_0$$

- Reduced Fast Subsystem

$$\mathbf{e}' = \mathbf{0}$$

$$\xi' = \mathbf{L}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \mathbf{z}_r') + \mathbf{K}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)(\mathbf{u}_0 + \mathbf{u}_f)$$

Step 2. Design Controller for Slow Subsystem

- Force origin as the equilibrium of the closed-loop slow subsystem

$$\begin{bmatrix} \mathbf{G}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r) \\ \mathbf{K}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r) \end{bmatrix} \mathbf{u}_0 = -\begin{bmatrix} \mathbf{F}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\dot{\mathbf{x}}_r) \\ \mathbf{L}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\mathbf{0}) \end{bmatrix} + \begin{bmatrix} A_{\mathbf{e}}\mathbf{e} \\ A_\xi \xi \end{bmatrix}$$

$$\dot{\mathbf{e}} = \mathbf{F}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\dot{\mathbf{x}}_r) + \mathbf{G}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r)\mathbf{u}_0$$

$$\mathbf{0} = \mathbf{L}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\mathbf{0}) + \mathbf{K}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r)\mathbf{u}_0$$

$$\dot{\mathbf{e}} = A_e \mathbf{e}$$

$$\mathbf{0} = A_\xi \xi$$

Resulting Closed-Loop Slow Subsystem

Step 3. Design Controller for Fast Subsystem

- To make sure the manifold is stable for all time,

$$\begin{bmatrix} \mathbf{G}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r) \\ \mathbf{K}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r) \end{bmatrix} \mathbf{u}_f = \begin{bmatrix} \mathbf{0} \\ \mathbf{L}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\mathbf{0}) - \mathbf{L}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\mathbf{z}_r') \end{bmatrix}$$

↓

$$\mathbf{e}' = \mathbf{0}$$

$$\xi' = \mathbf{L}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r,\mathbf{z}_r') + \mathbf{K}(\mathbf{e},\xi,\mathbf{x}_r,\mathbf{z}_r)(\mathbf{u}_0 + \mathbf{u}_f)$$

↓

$$\mathbf{e}' = \mathbf{0}$$

$$\xi' = A_\xi \xi$$

Resulting Closed-Loop Fast Subsystem

Step 4. Composite Control Design

- Composite Control: $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_f$

- Or, $$\begin{bmatrix} \mathbf{G}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r) \\ \mathbf{K}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r) \end{bmatrix} \mathbf{u} = -\begin{bmatrix} \mathbf{F}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r,\dot{\mathbf{x}}_r) \\ \mathbf{L}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r,\mathbf{z}_r') \end{bmatrix} + \begin{bmatrix} A_e \mathbf{e} \\ A_\xi \boldsymbol{\xi} \end{bmatrix}$$

Resulting Closed-Loop System

$$\dot{\mathbf{e}} = \mathbf{F}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r,\dot{\mathbf{x}}_r) + \mathbf{G}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r)\mathbf{u}$$
$$\epsilon\dot{\boldsymbol{\xi}} = \mathbf{L}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r,\epsilon\dot{\mathbf{z}}_r) + \mathbf{K}(\mathbf{e},\boldsymbol{\xi},\mathbf{x}_r,\mathbf{z}_r)\mathbf{u}$$

$$\dot{\mathbf{e}} = A_e \mathbf{e}$$
$$\epsilon\dot{\xi} = A_\xi \xi$$

Lyapunov Analysis

- Lyapunov Function Candidate:

$$v(\mathbf{e},\zeta)=(1-d)\mathbf{e}^T\mathbf{e}+d\zeta^T\zeta,\quad d>0$$

- Time derivative about closed-loop dynamics:

$$\dot{v}\le-\begin{bmatrix}\mathbf{e}\\ \xi\end{bmatrix}^T\begin{bmatrix}(1-d)(\alpha_1-2\alpha) & 0\\ 0 & \frac{d}{\epsilon}\alpha_2-2\alpha d\end{bmatrix}\begin{bmatrix}\mathbf{e}\\ \xi\end{bmatrix}-2\alpha v$$

$$\alpha_1=2|\lambda_{\min}(A_e)|,\quad \alpha_2=2|\lambda_{\min}(A_\xi)|,\quad \alpha>0$$

- If $\epsilon<\dfrac{\alpha_2}{2\alpha}$, $\quad \dot{v}\le-2\alpha v$
- Or, $\quad \dot{v}\le-2\alpha\gamma_{11}\left\|\begin{bmatrix}\mathbf{e}\\ \xi\end{bmatrix}\right\|^2$
- Thus, global exponential stability can be concluded.
- Or $\quad \mathbf{x}(t)\to\mathbf{x}_r(t),\ \mathbf{z}(t)\to\mathbf{z}_r(t), t\to\infty$

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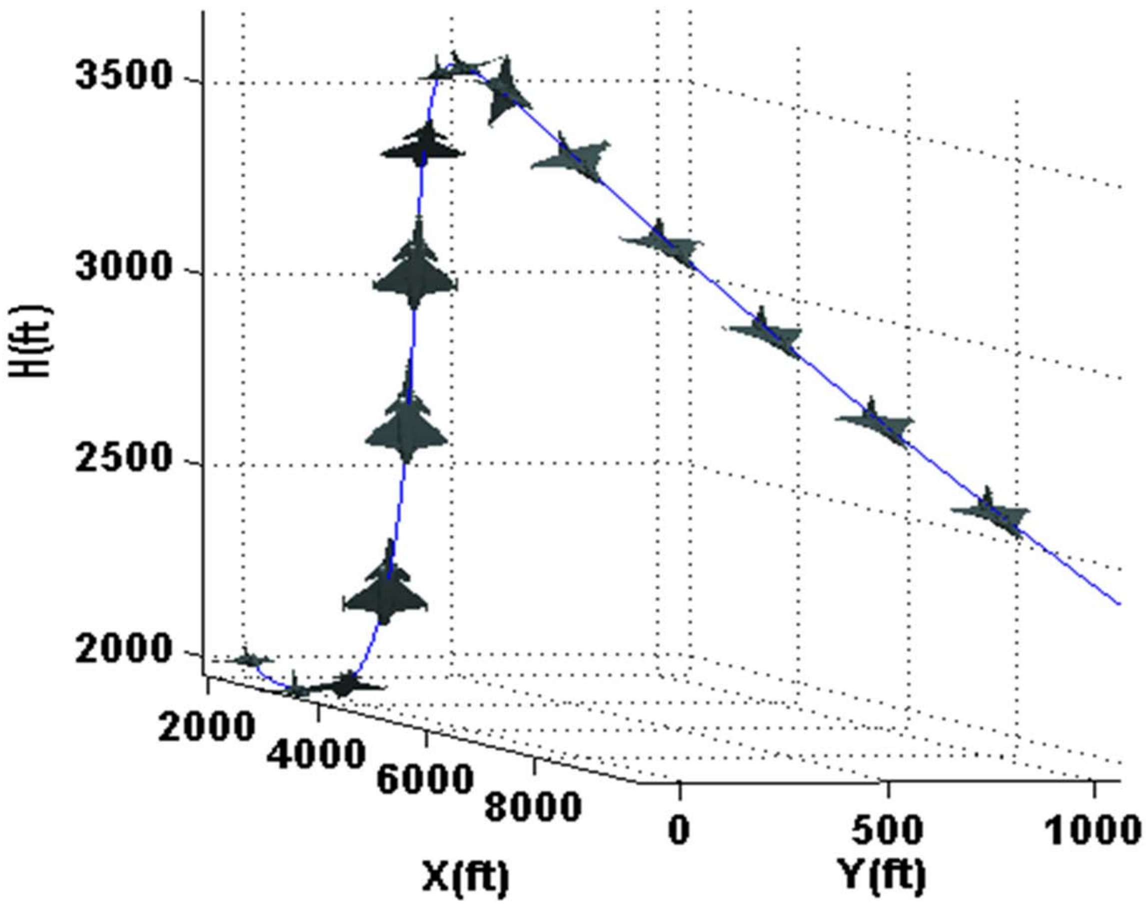
Numerical Example

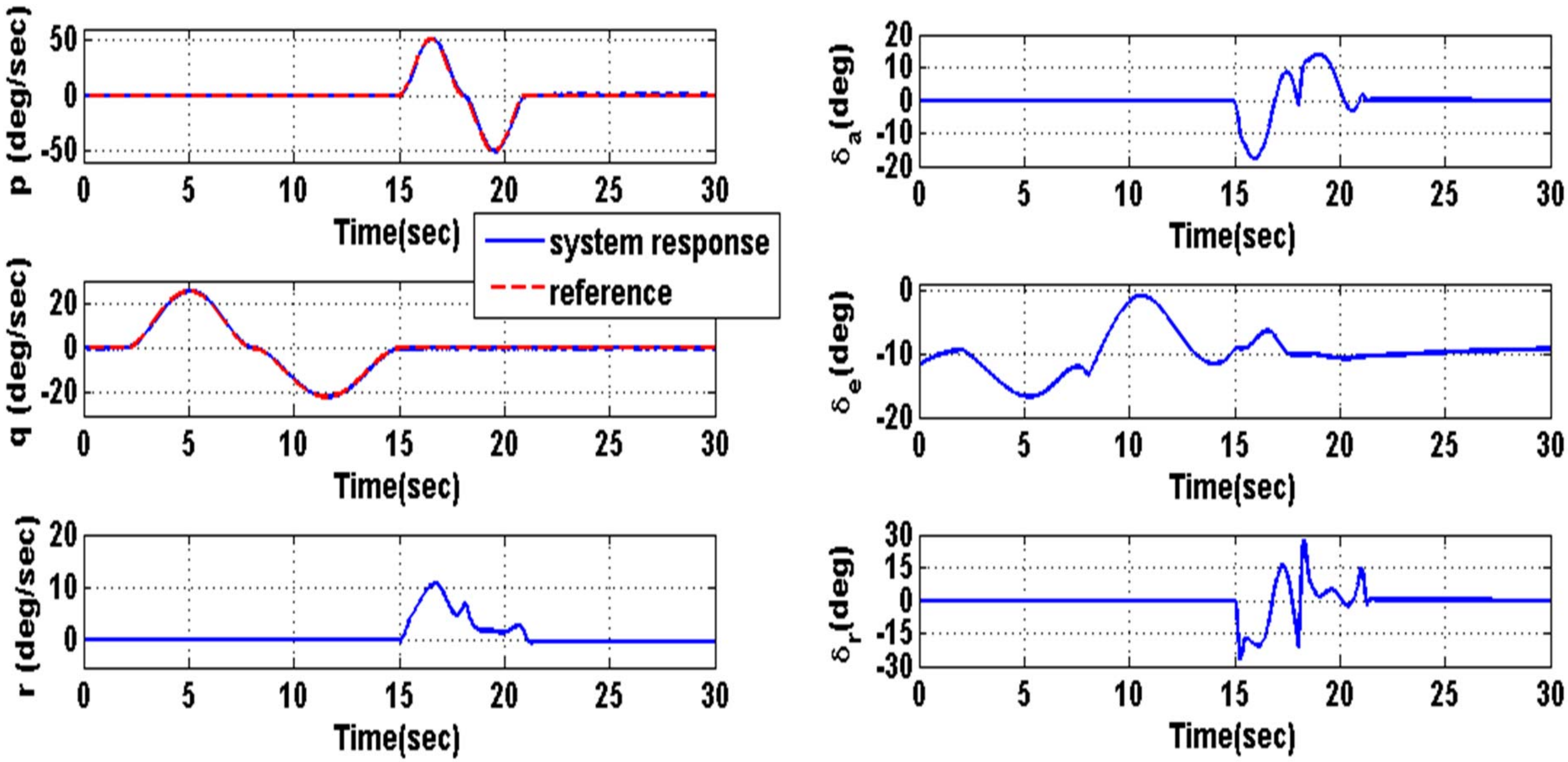
- Nonlinear, six degree-of-freedom F/A-18A Hornet written in the stability axis.

- Slow states: $\mathbf{x} = [M, \alpha, \beta, \phi, \theta, \psi]^T$
- Fast states: $\mathbf{z} = [p, q, r]^T$
- Control Variables: $u = [\eta, \delta_e, \delta_a, \delta_r]^T$
- Objective: Aggressive vertical climb with maximum pitch rate of 25deg/sec followed by roll at a rate of 50deg/sec with sideslip angle constrained to zero throughout.

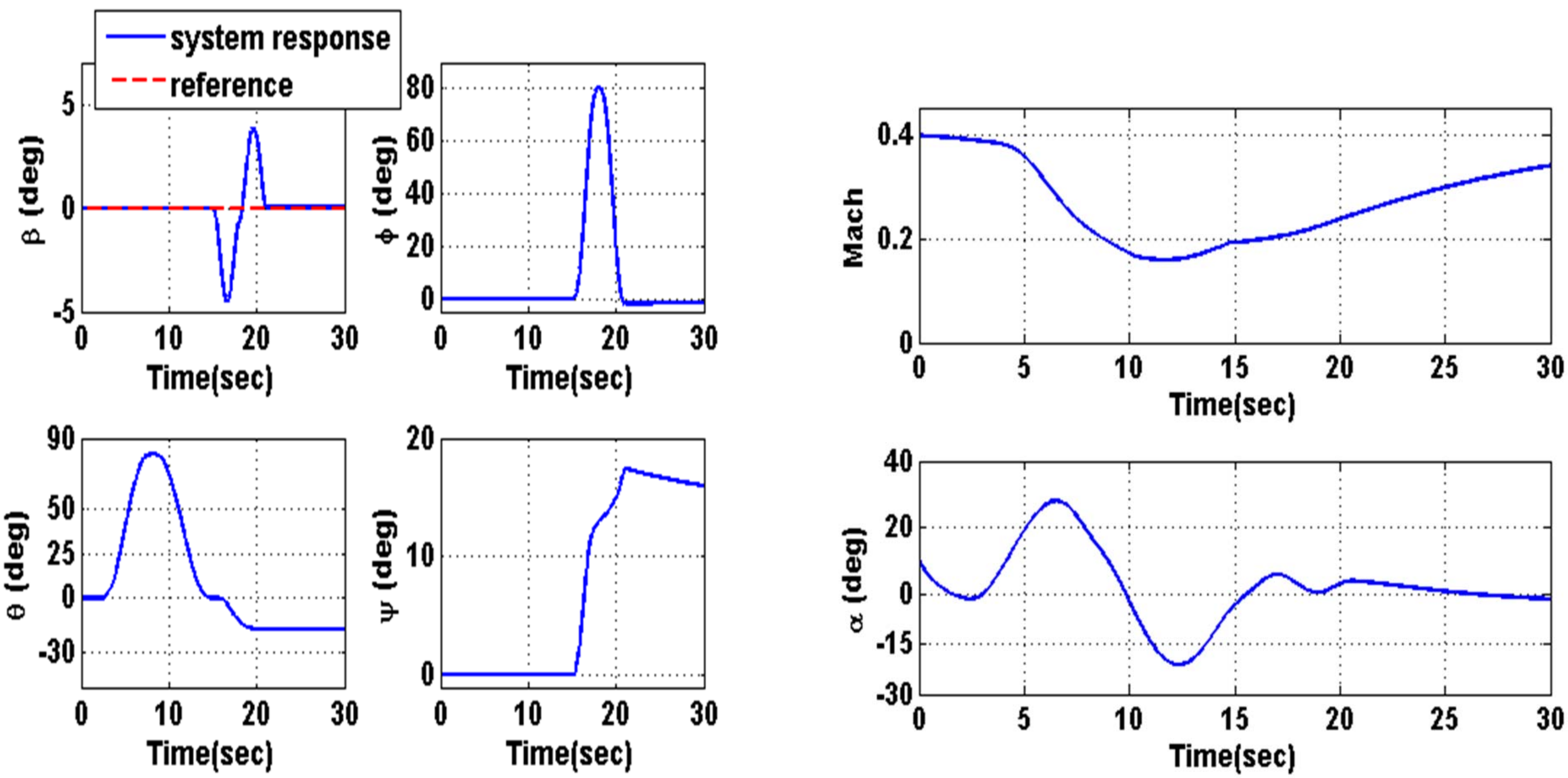
Aircraft Trajectory



Angular Rates and Control Deflections



Response of Slow State States



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Summary and Conclusions

- Developed tracking controllers to simultaneously track slow and fast states for **nonlinear singularly perturbed system** using geometric singular perturbation theory as a model-reduction technique
- Controller is implemented without making any assumptions about the nonlinear model and does not require knowledge of the perturbation parameter.
- Consistent tracking guaranteed independent of the reference trajectory.
- Global asymptotic tracking demonstrated even though the reference trajectory requires airplane to switch between linear and nonlinear regimes.
- All closed-loop signals were bounded and control deflections computed were smooth.

Future Research Extensions

- Global stabilization without tracking fast states
  - How can global results be asserted? (Siddarth & Valasek, ACC 2011)
- Extensions for systems nonlinear in control
  - Current work assumes the control appears linearly

$$\dot{\mathbf{e}} = \mathbf{F}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \dot{\mathbf{x}}_r) + \mathbf{G}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)\mathbf{u}$$

$$\epsilon\dot{\xi} = \mathbf{L}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r, \epsilon\dot{\mathbf{z}}_r) + \mathbf{K}(\mathbf{e}, \xi, \mathbf{x}_r, \mathbf{z}_r)\mathbf{u}$$

- Non-minimum phase system control architecture
  - Take advantage of the multiple-time scale behaviour of aircraft (Siddarth & Valasek, AIAA GNC 2011)
- Autonomous control of Highly Reconfigurable Structures
  - Adaptive-Reinforcement Learning (Valasek et.al, 2004, 2006)

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QUESTIONS?