Autopilot for a Nonlinear Non-Minimum Phase Tail-Controlled Missile

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Research Motivation

Control challenges for high-agile tail controlled missiles:

1. Exploiting the full physical capabilities of the missile system
   - Fulfilling demanding performance requirements
   - Exploiting the complete flight envelope *without exciting the unstable internal dynamics*

2. Coping with a large spectrum of uncertainties and unknown dynamics
   - Identifying the missile aerodynamics is expensive and inaccurate
   - Only certain states can be measured
Acceleration Control of a Tail-Controlled Missile

**Introduction**

**The Issues**

**The Solution**

**Findings**

**Conclusions**

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**Basic setup:**
The autopilot is turned on when the missile is flying at steady-state horizontal flight.

**Description:**
Planar longitudinal dynamics
Fin-controlled

**Control objective:**
Follow a desired normal acceleration command

**Features:**
Inherently statically stable in pitch
Aft control surface causes non-minimum phase behavior in the longitudinal motion
Simulation Model

**Introduction**

The issues leading to the simulation model are...  

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**The Solution**

The solution involves modeling the actuator system and the sensor system as follows:

- **Actuator System:**
  - Actuator modeled as a second order system with limits in deflection, rate and acceleration.

- **Sensor System:**
  - IMU located at the c.g and modeled as a first order system. Measurements are angular rate, linear accelerations.

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**Rigid-body equations of motion:**

$$
\dot{\mathbf{v}}_k^G = \frac{1}{m} \left[ \cos \alpha_k^G \left( F_X^G \right)_B + \sin \alpha_k^G \left( F_Z^G \right)_B \right],
$$

$$
\dot{\alpha}_k^G = \dot{q}_k^{oB} + \frac{1}{v_k^G} \left[ \frac{1}{m} \left\{ \cos \alpha_k^G \left( F_Z^G \right)_B - \sin \alpha_k^G \left( F_X^G \right)_B \right\} \right],
$$

$$
\dot{\theta} = \dot{q}_k^{oB},
$$

$$
\ddot{q}_k^{oB} = \frac{1}{I_{\gamma B}} (M_T^G)_B.
$$

---

**Symbols:**

- $\alpha_k^G$: angle-of-attack
- $q_k^{oB}$: body-axis angular rate
- $\delta_M$: fin deflection

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**Findings**

The findings of the simulation model are...
Simulation Model

Realistic Aerodynamic Data Set

Parametric Uncertainties

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty Range</th>
<th>Units</th>
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<tr>
<td>$I_{YB}$</td>
<td>±5%</td>
<td>[-]</td>
</tr>
<tr>
<td>$m$</td>
<td>±1%</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_{cg}$</td>
<td>±50 [mm]</td>
<td></td>
</tr>
<tr>
<td>$C_{x,0}, C_{z,0}$</td>
<td>±10%</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{m,0}$</td>
<td>±20%</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{x,\delta M}, C_{z,\delta M}, C_{m,\delta M}$</td>
<td>±20%</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{m,q}$</td>
<td>±20%</td>
<td>[-]</td>
</tr>
<tr>
<td>$\alpha_{K}$</td>
<td>±2.5 [deg]</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
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<td>[-]</td>
</tr>
<tr>
<td>$\bar{q}$</td>
<td>±5%</td>
<td>[-]</td>
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# Problem Description

**Control objective:**
Generate a control law for fin deflection $\delta_M$ that produces the desired normal acceleration while ensuring closed-loop stability, or $y \to y_r$ as $t \to \infty$

**Assumptions:**
- Velocity is bounded for all time (example: endgame scenario)
- Autopilot turned on when the missile is flying steady-state horizontal flight
- Kinematic orientation angle (pitch attitude angle) ignored in design

**Reduced dynamical model:**
\[
\begin{align*}
\dot{\alpha}_K^G &= c_0 + c_1 q_{KB}^O + c_2 \delta_M, \\
\dot{q}_{KB}^O &= c_3 + c_4 q_{KB}^O + c_5 \delta_M, \\
y &= c_6 + c_7 q_{KB}^O + c_8 \delta_M.
\end{align*}
\]

**Key benefit:** autopilot independent of the velocity
Non-Minimum Phase Behaviour

**Exact Input-Output Linearization:**
Relative degree = 1 (since fin deflection contribution to normal force is small)

\[
\dot{y} = \dot{c}_6 + \dot{c}_7q_K^{OB} + c_7q_K^{OB} \\
= \dot{c}_6 + \dot{c}_7q_K^{OB} + c_7(c_3 + c_4q_K^{OB} + c_5\delta_M) \\
\dot{\alpha}_K^G = c_0 + c_1q_K^{OB} + c_2\delta_M
\]

Strong angle-of-attack & angular rate coupling is the source of divergence
## Previous Work

1. **Gain-scheduling**
   - *Limits performance and maneuverability*

2. **Modify the problem (place the sensor ahead of the center of rotation)**
   - *Location of the center of rotation changes substantially as the vehicle travels between transonic and supersonic regime*

3. **Ignore the angle-of-attack and pitch rate coupling (approximate input-output linearization)**
   - *Approximate tracking performance, no quantitative estimate of the error can be given*

4. **Feedfoward control**
   - *Reference specific; offline-strategy*
Autopilot Design

Issues:
- Approximate tracking if the coupling is ignored
- Non-minimum phase if the coupling is retained
- No real-time, nonlinear, reference independent solutions
- Uncertain aerodynamics

Approach:
- Employ the coupling to synthesize the controller via a cascaded approach

Benefits:
- Guaranteed asymptotic output tracking for the nonlinear system
- Independent of and can handle time-varying desired reference trajectory
- No need for mission-dependent tuning of feedback gains
- Does not require inherent time-scale separation
The Approach

Develop closed-loop such that internal state trajectory is computed online

**Step 1:** Given the desired normal acceleration command, determine the sufficient angular rate command required to change the angle-of-attack appropriately.

**Step 2:** Using this knowledge determine the control deflection command to generate the appropriate pitching moment

**Main contribution:** Formulation of design criteria that guarantees the approach can be implemented even when the vehicle has no time scale separation
**Step 1: Compute the desired internal state trajectory**

\[
\dot{y} = c_6 + c_7 q^0 + c_8 \delta_M (\alpha^K_G, q^0) + c_8 \dot{\delta}_M (\alpha^K_G, q^0)
\]

**Assign:**

\[
\dot{y} = \bar{v}
\]

where the virtual control input

\[
\bar{v} = K_v x
\]

**with**

\[
\dot{x} = -K_p (K_v x - \dot{y}_r) - K_I (y - y_r)
\]

and \(K_p, K_I\) are positive feedback gains.

**Determine** \(q^0(\alpha)\) **using:**

\[
\frac{\partial c_6}{\partial \alpha^K_G} \left[ c_0 + c_i q^0 + \left( \frac{b_x c_9}{V^K_G} \cos \alpha^K_G - c_{i0} \right) \delta_M (\alpha^K_G, q^0) \right] = \bar{v}
\]
Step 2: Stabilize angular rate

Formulate the fin deflection control law

\[ \delta_M = - \frac{K_q (q_K^{oB} - q) + c_3 + c_4 q_K^{oB}}{c_5} \]

to assign closed-loop pitch rate dynamics:

\[ \dot{q}_K^{oB} = -K_q (q_K^{oB} - q) \]

Control Law Summary:

Given reference trajectory \( y_r \)

(i) time-varying pitch rate manifold is computed using

\[ \frac{\partial c_6}{\partial \alpha_K^{G}} \left[ c_0 + c_1 q^o + c_{10} \left( \frac{c_3 + c_4 q^o}{c_5} \right) \right] = \bar{V} \]

(ii) which is used in

\[ \delta_M = - \frac{K_q (q_K^{oB} - q^o) + c_3 + c_4 q_K^{oB}}{c_5} \]

to get commanded fin deflection
Closed-Loop Analysis

Objective: Develop a design criterion for selecting the feedback gains that ensures:
1. the closed-loop system is asymptotically stable and
2. output follows the desired reference
3. under parametric and aerodynamic uncertainties

\[
\hat{\alpha}_k^G = \frac{K_v x}{\partial c_6} + c_1 (q_{K}^{oB} - q^o) + K_q \frac{c_{10}}{c_5} (q_{K}^{oB} - q^o) + \epsilon \frac{c_9}{V_{K}^G} \cos \alpha_k^G \delta_M, \notag
\]

\[
\dot{q}_{K}^{oB} = -K_q (q_{K}^{oB} - q^o), \notag
\]

\[
\dot{x} = -K_p (K_v x - \dot{y}_r) - K_I \left( c_6 + c_7 q_{K}^{oB} + \epsilon c_9 \delta_M - y_r \right). \notag
\]
Closed-Loop Analysis

**Trim \((\alpha^*, q^*, x^*)\) Equations:**

\[
\begin{align*}
q^* &= q^0(\alpha^*), \\
\frac{K_v x^*}{\partial c_6} + \epsilon \frac{c_9}{V^G_K} \cos \alpha M(\alpha^*, q^*) &= 0,
\end{align*}
\]

Closed-loop system with origin as the unique equilibrium:

\[
\begin{align*}
\dot{\alpha}^G_K &= \left[ \frac{K_v x^*}{\partial c_6} - \frac{K_v x^*}{\partial c_6} \right] + \left[ \frac{K_q}{c_{10}} \right] \left( q^B_0 - q^* \right) + \\
&+ \left[ \frac{c_9}{V^G_K} \cos \alpha (\alpha^*) \delta_M(\alpha^*, q^*) \right] + \left[ \frac{c_1 + K_q}{c_5} \right] \left( q^* - q^0(\alpha^G_K) \right), \\
\dot{q}^B_0 &= -K_q (q^B_0 - q^*) - K_q (q^* - q^0(\alpha^G_K)), \\
\dot{x} &= -K_p \Delta x - K_I \left\{ c_6 - c_6(\alpha^*) \right\} + \left\{ c_4 q^B_0 - c_7(\alpha^*) q^* \right\} + \left\{ c_9 \delta_M - c_9(\alpha^*) c_9 \delta_M(\alpha^*, q^*) \right\}.
\end{align*}
\]
Lyapunov Based Analysis

Lyapunov function candidate: \[ V = \frac{1}{2} (x - x^*)^2 + \frac{1}{2} (\alpha^G_K - \alpha^*)^2 + \frac{1}{2} (q^0_K - q^*)^2 \]

Lyapunov derivative:
\[
\dot{V} = -K_v K_p (1/c_g) (x - x^*)^2 - K_q (q^0_K - q^*)^2 - K_q (q^* - q^0 (\alpha^G_K))(q^0_K - q^*) \\
- (1/c_g) K_I \left[ \left\{ c_6 - c_6(\alpha^*) \right\} + \left\{ c_7 q^0_K - c_7 (\alpha^*) q^* \right\} + \left\{ \epsilon c_9 \delta_M - \epsilon(\alpha^*) c_9 \delta_M (\alpha^*, q^*) \right\} \right] (x - x^*) \\
\left[ \frac{K_v x}{\partial c_6}, \frac{K_v x^*}{\partial c_6}, \frac{\partial \alpha^G_K}{\partial \alpha^*} \right] (\alpha^G_K - \alpha^*) + \left[ c_1 + K_q \frac{c_{10}}{c_5} \right] (\alpha^G_K - \alpha^*) (q^0_K - q^*) \\
+ \left[ \epsilon \frac{c_9}{\nu^G_K} \cos \alpha^G_K \delta_M - \epsilon(\alpha^*) \frac{c_9}{\nu^G_K} \cos \alpha^* \delta_M (\alpha^*, q^*) + \left\{ c_1 + K_q \frac{c_{10}}{c_5} \right\} (q^* - q^0 (\alpha^G_K)) \right] (\alpha^G_K - \alpha^*)
\]

Feedback gains can be selected offline using Monte Carlo simulations but instead here we use analytic interconnection condition evaluation
Interconnection Conditions

Due to quadratic nature of terms:

\[(1/c_9)K_v K_p (x - x^*)^2 = (1/c_9)K_v K_p |(x - x^*)|^2\]

\[K_q (q_K^{oB} - q^*)^2 = K_q |(q_K^{oB} - q^*)|^2\]

Using triangle inequality, nature of aerodynamic coefficients,

\[K_I \left[ \left\{ c_6 - c_6(\alpha^*) \right\} + \left\{ c_7 q_K^{oB} - c_7(\alpha^*)q^* \right\} + \left\{ \epsilon c_9 \delta_M - \epsilon(\alpha^*)c_9 \delta_M(\alpha^*, q^*) \right\} \right] (x - x^*) \leq\]

\[-K_I c_9 \left| s_{b_0} \right| \left| (\alpha_K^G - \alpha^*) \right| \left| (x - x^*) \right| - K_I K_q c_9 \left| \epsilon \right| \left| (q_K^{oB} - q^*) \right| \left| (x - x^*) \right|\]

\[+K_I c_9 \left| b_1(\alpha_K^G) \left\{ q_K^{oB} - q^* \right\} \right| \left| (x - x^*) \right| - K_I c_9 \beta \left| q^* \right| \left| \alpha_K^G - \alpha^* \right| \left| (x - x^*) \right|\]

Other inequalities are given in the paper.
Lyapunov Based Analysis

Lyapunov function candidate:  \[ V = \frac{1}{2} (x - x^*)^2 + \frac{1}{2} (\alpha_K^G - \alpha^*)^2 + \frac{1}{2} (q_K^0 - q^*)^2 \]

Lyapunov derivative:
\[ \dot{V} \leq \left[ (x - x^*) \ (\alpha_K^G - \alpha^*) \ (q_K^0 - q^*) \right] \mathbb{K} \left[ \begin{array}{c} (x - x^*) \\ (\alpha_K^G - \alpha^*) \\ (q_K^0 - q^*) \end{array} \right] \]
\[ \mathbb{K} = \begin{bmatrix} -K_v K_p (1 / c_9) & \mu_1 & \mu_2 \\ \mu_1 & -\mu_\alpha & \mu_3 \\ \mu_2 & \mu_3 & -K_q \end{bmatrix} \]

Thus, feedback gains must be selected to ensure the above matrix is negative-definite for a range of parametric uncertainties.

Can be shown that such a feedback gain choice makes the closed-loop system singularly perturbed
Case Study

**Objective:** To demonstrate controller performance in simulation and illustrate how to employ matrix $K$ to choose feedback gains

**Flight condition:**

$V_k^G = 600m/\text{sec}$

$h = 5km$

**Feedback gain selection:**

1. Use least upper-bounds for aerodynamic terms in the matrix $K$.
2. Pick the gains $K_v, K_\rho, K_I$ to assign desired dynamics to the output

\[
\frac{y(s)}{y_r(s)} = \frac{K_v[K_\rho s + K_I]}{s^2 + K_vK_\rho s + K_vK_I}
\]

Then, using this initial guess, iterate the gain for angular rate to ensure matrix has desired properties.

Found that the integral action propagates itself into the matrix making $K$ negative semi-definite. **BUT asymptotic guarantees can still be proven.**
Case Study: Simulation Results
Conclusions and Future Recommendations

Non-minimum phase properties addressed by inducing a time scale separation

Missile dynamics is not approximately feedback linearizable (cannot ignore the small influence terms that make the control problem minimum phase)

Autopilot design is independent of the desired reference and causal

Final journal paper will study the implications of inducing such a time scale separation and provide robustness analysis results.
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