MODEL PREDICTIVE VARIABLE STRUCTURE CONTROL WITH MODEL FOLLOWING FOR FOREBODY VORTEX FLOW CONTROL

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WHAT IS THE COMMAND GENERATOR TRACKER?

The Command Generator Tracker commands system outputs to track time-varying dynamic model outputs.

FEATURES

- By using a desired set of non-stationary model dynamics, time-varying model outputs can be tracked with a feedforward structure.
  - Capability to make the dynamics of one aircraft mimic the dynamics of a different aircraft
- The tracking dynamics not only have zero steady-state error, but also force the system dynamics to emulate the model dynamics.
MODEL-FOLLOWING CGT development (1)

Given the dynamic system and output:

\[ x_{k+1} = \Phi x_k + \Gamma u_k \]
\[ y_k = H x_k + D u_k \]

and CGT model and output:

\[ x_{m,k+1} = \Phi_m x_{m,k} + \Gamma_m u_{m,k} \]
\[ y_{m,k} = H_m x_{m,k} + D_m u_{m,k} \]

Find: \( u_k \) that will drive \( y_k^* \) to \( y_m \) as \( t \to \infty \) and remain there.

In steady state when \( y_k^* = y_m \), then the state and controls will be at \( x_k^* \) and \( u_k^* \). Assume a relationship between the model and the system steady state of the form:

\[
\begin{bmatrix}
    x_k^* \\
    u_k^*
\end{bmatrix} =
\begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
    x_{m,k} \\
    u_{m,k}
\end{bmatrix}
\]
MODEL-FOLLOWING CGT development (2)

In steady state the plant and the output satisfy:

\[ x_{k+1}^* = \Phi x_k^* + \Gamma u_k^* \quad \text{and} \quad y_k^* = H x_k^* + D u_k^* \]

Construct and expression for \( x_{k+1}^* - x_k^* \) and combine it with \( y_k^* \):

\[
\begin{bmatrix}
    x_{k+1}^* - x_k^* \\
    y_k^*
\end{bmatrix} = \begin{bmatrix}
    (\Phi - I) & \Gamma \\
    H & D
\end{bmatrix} \begin{bmatrix}
    x_k^* \\
    u_k^*
\end{bmatrix}
\]

Also, from the assumed solution form:

\[ x_k^* = A_{11} x_{m,k} + A_{12} u_{m,k} \]

Forming an expression for \( x_{k+1}^* - x_k^* \) using the above relation and combining it with the system output equation:

\[
\begin{bmatrix}
    x_{k+1}^* - x_k^* \\
    y_k 
\end{bmatrix} = \begin{bmatrix}
    A_{11} (x_{m,k+1} - x_{m,k}) + A_{12} (u_{m,k+1} - u_{m,k}) \\
    H_m x_{m,k} + D_m u_{m,k}
\end{bmatrix}
\]
MODEL-FOLLOWING CGT development (3)

Substituting for \( x_{m,k+1} \):

\[
\begin{bmatrix}
    x_{k+1}^* - x_k^* \\
    y_k
\end{bmatrix} =
\begin{bmatrix}
    A_{11} \left[ (\Phi - I)x_{m,k} + \Gamma_m u_{m,k} \right] + A_{12} \left( u_{m,k+1} - u_{m,k} \right) \\
    H_m x_{m,k} + D_m u_{m,k}
\end{bmatrix}
\]

Assume that \( u_{m,k+1} = u_{m,k} = u_m \), i.e., the model control is a constant over a sample period, and applying the desired condition \( y_k^* = y_m \), results in:

\[
\begin{bmatrix}
    (\Phi - I) & \Gamma \\
    H & D
\end{bmatrix}
\begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix}
= 
\begin{bmatrix}
    A_{11} (\Phi_m - I) & A_{11} \Gamma_m \\
    H_m & D_m
\end{bmatrix}
\]

Introducing the notation for the inverse of the discrete Quad Partition Matrix gives:

\[
\begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix} = 
\begin{bmatrix}
    \pi_{11} & \pi_{12} \\
    \pi_{21} & \pi_{22}
\end{bmatrix}
\begin{bmatrix}
    A_{11} (\Phi_m - I) & A_{11} \Gamma_m \\
    H_m & D_m
\end{bmatrix}
\]
Rewriting this relationship as individual equations:

\[
A_{11} = \pi_{11} A_{11} (\Phi_m - I) + \pi_{12} H_m
\]

\[
A_{12} = \pi_{11} A_{11} \Gamma_m + \pi_{12} D_m
\]

\[
A_{21} = \pi_{21} A_{11} (\Phi_m - I) + \pi_{22} H_m
\]

\[
A_{22} = \pi_{21} A_{11} \Gamma_m + \pi_{22} D_m
\]

**OBSERVATIONS**

1. The first equation is a non-symmetric Lyapunov Matrix equation with \(A_{11}\) the only unknown.
2. Once \(A_{11}\) is known, the other unknowns can be solved by evaluation.
The optimal controller has the form:

\[ u_k = (u_k^* + Kx_{k}^*) - Kx_k \]

In the CGT development \( u^* \) and \( x^* \) are given by:

\[
\begin{bmatrix}
  x_k^* \\
  u_k^*
\end{bmatrix} =
\begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
  x_{m,k} \\
  u_m
\end{bmatrix}
\]

Therefore, the CGT optimal controller can be written as:

\[ u_k = u_k^* - K(x_k - x_k^*) \]

Substituting for \( u^* \) and \( x^* \) and collecting terms:

\[ u_k = [(A_{21} + KA_{11})x_{m,k} + (A_{22} + KA_{12})u_m] - Kx_k \]
MODEL-FOLLOWING CGT
Selections of the models

- THE SELECTIONS OF THE COMMAND GENERATOR AIRCRAFT AND THE TRACKER AIRCRAFT ARE UNRESTRICTED*.
  - Two different aircraft
    - Train F-18 pilots using T-38
  - The same aircraft at different flight conditions
    - Flying quality improvements
MODEL-FOLLOWING CGT restrictions and special case

- **RESTRICTIONS**
  - Both the model and tracker dynamics must be observable and controllable.
  - The maximum number of model outputs that can be tracked is equal to the number of available control inputs.
  - The plant has to have faster dynamics than that of the model for good tracking.

- **SPECIAL CASE**
  - If the model dynamics are stationary, CGT reduces to the Nonzero Setpoint (NZSP) structure.
  - The NZSP is capable of tracking only constant command inputs.
MODEL-FOLLOWING DESIGN

- **OBJECTIVE**
  - Design a closed-loop lateral/directional controller that will make the high angle-of-attack system model (the command generator) track the trajectory of the low angle-of-attack system (the tracker)

- **TRAJECTORY PARAMETERS**
  - Sideslip angle and bank angle of the command generator aircraft are selected as the trajectory parameters to be tracked by the tracker aircraft.

- **APPROACH**
  - The maneuver of the generator aircraft is commanded by the Nonzero Setpoint (NZSP) structure.
  - *Model-following Command Generator Tracker (CGT) is employed to make the tracker aircraft follow the selected trajectory parameters of the command generator aircraft.*
SIMULATION STRUCTURE

STATIONARY COMMAND → NONZERO SETPOINT → COMMAND GENERATOR AIRCRAFT

COMMAND GENERATOR AIRCRAFT → CGT

CGT → TRACKER AIRCRAFT

TRACKER AIRCRAFT → NON-STATIONARY COMMAND
SIMULATION MODELS

**FLIGHT CONDITIONS**

\[ M = 0.35 \quad h = 38,000 \text{ ft} \]
\[ \alpha = 40^\circ \quad V = 338 \text{ ft/sec} \]
\[ \bar{q} = 37 \text{ psf} \quad \delta_{\text{canard}} = -24.4^\circ \]
\[ \delta_{\text{strake}} = 12.8^\circ \quad \delta_{\text{symmetric flap}} = 20.7^\circ \]

**LINEAR MODEL**

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{\rho} \\
\dot{r} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
0.043 & 0.64 & -0.77 & 0.073 \\
-8.39 & 0.78 & -0.69 & -7.4 \times 10^{-5} \\
0.14 & 0.063 & -0.11 & -1.7 \times 10^{-5} \\
0 & 1 & 0.84 & 0
\end{bmatrix}
\begin{bmatrix}
\beta \\
\rho \\
r \\
\phi
\end{bmatrix}
\]
\[
+ \begin{bmatrix}
-0.0034 & 0.0013 \\
0.86 & -1.17 \\
0.092 & -0.12 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_u \\
\delta_r \\
\delta_y \\
\delta_w
\end{bmatrix}
\]

**FLIGHT CONDITIONS**

\[ M = 0.70 \quad h = 20,000 \text{ ft} \]
\[ \alpha = 4.10^\circ \quad V = 726 \text{ ft/sec} \]
\[ \bar{q} = 334 \text{ psf} \quad \delta_{\text{canard}} = -2.93^\circ \]
\[ \delta_{\text{strake}} = -3.87^\circ \quad \delta_{\text{symmetric flap}} = 0.87^\circ \]

**LINEAR MODEL**

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{\rho} \\
\dot{r} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
-0.16 & 0.072 & -1.0 & 0.044 \\
-15 & -2.6 & 1.1 & 0 \\
6.8 & -0.10 & -0.064 & 0 \\
0 & 1 & 0.072 & 0
\end{bmatrix}
\begin{bmatrix}
\beta \\
\rho \\
r \\
\phi
\end{bmatrix}
\]
\[
+ \begin{bmatrix}
-0.034 & 0.039 \\
77 & 13 \\
5.1 & -4.1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_u \\
\delta_r \\
\delta_y \\
\delta_w
\end{bmatrix}
\]
SIMULATION CASES

■ CASE 1
- Generator: High-\(\alpha\) X-29A
- Tracker: Low-\(\alpha\) X-29A
- Commands:
  \(\beta = 0^\circ \Rightarrow -1^\circ \Rightarrow +1^\circ\)
  \(\phi = 0^\circ \Rightarrow -5^\circ \Rightarrow +5^\circ\)

■ CASE 2
- Generator: Low-\(\alpha\) X-29A
- Tracker: High-\(\alpha\) X-29A
- Commands:
  \(\beta = 0^\circ \Rightarrow -1^\circ \Rightarrow +1^\circ\)
  \(\phi = 0^\circ \Rightarrow -5^\circ \Rightarrow +5^\circ\)
CASE 1: Generator: High $\alpha$, Tracker: Low $\alpha$
Case 2: Generator: Low $\alpha$, Tracker: High $\alpha$
HIGH $\alpha$ MANEUVERING

AGILITY OF FIGHTER TYPE AIRCRAFT AT HIGH ANGLE-OF-ATTACK CAN BE SERIOUSLY DEGRADED BECAUSE OF THE FOLLOWING FACTORS:

1. Unstable pitch break
2. Ineffective propulsion/airframe integration
3. Loss of static and dynamic lateral/directional stability
4. Reduced control effectiveness

TYPICAL YAW CONTROL REQUIREMENTS FOR MANEUVERING
X-29A aircraft has been modified to test the effectiveness of the Pneumatic Vortex Nozzles (PVC).

Vortex flow control involves pneumatic manipulation of forebody vortices.

As the diagram shows, air exhaust through the right nozzle accelerates the flow of the right vortex and pulls it closer to the forebody.

As this occurs, the left vortex is pushed further away from the body. This results in lower pressure on the side of the blowing right nozzle, resulting in a right yawing movement of the aircraft as shown.
The available lateral control inputs are:
- Differential flaps and Rudder (continuous)
- VFC nozzles (bang-off-bang)

The MPVSC calculates three different values of cost forward in time at the beginning of each sample period using the following quadratic cost function. Each cost value corresponds to the three possible states of the VFC nozzle control.

\[
J = \frac{1}{2} \int_{0}^{t_f} \{X^T Q X + U^T R U \} \, dt
\]

Then, the MPVSC selects the VFC nozzle control state which generates the lowest cost for the upcoming sample period.
MODEL PREDICTIVE VARIABLE STRUCTURE CONTROLLER (2)

Block Diagram Representation of MPVSC Controller
SIMULATION CASES

■ CASE 3
- Generator: Low $\alpha$ X-29A
- Tracker: High $\alpha$ X-29A
- Commands:
  - $\beta = 4^\circ \Rightarrow -1^\circ$
  - $\phi = 10^\circ \Rightarrow -10^\circ$
- MPVSC: OFF

■ CASE 4
- Generator: Low $\alpha$ X-29A
- Tracker: High $\alpha$ X-29A
- Commands:
  - $\beta = 4^\circ \Rightarrow -1^\circ$
  - $\phi = 10^\circ \Rightarrow -10^\circ$
- MPVSC: ON
CASE 3: Generator: Low $\alpha$, Tracker: High $\alpha$; MPVSC: Off

Tracker cost = 2540
Case 4: Generator: Low $\alpha$, Tracker: High $\alpha$; MPVSC: On

Tracker cost = 1018
CONCLUSIONS

For the aircraft model and flight condition tested, the Model-Following Command Generator Tracker can permit an aircraft flying at a low speed, high angle-of-attack flight condition (with undesirable dynamics) to effectively track the time varying outputs of a high speed, low angle-of-attack aircraft model (with desirable dynamics).

Proper integration of the Model-Following Command Generator Tracker with an existing Model Predictive Variable Structure Controller can enhance the performance of the tracking in the flight conditions where an insufficient amount of the conventional aerodynamic control power is available.