Model Predictive Variable Structure Control with Model Following for Forebody Vortex Flow Control

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Aircraft which are required to fly effectively at high angles-of-attack must overcome two fundamental difficulties: generally poor dynamic characteristics, especially with respect to damping, and low control authority of conventional aerodynamic effectors. One solution to improve closed-loop tracking of pilot commands is to employ a model-following controller approach, whereby an aircraft with poor dynamics at high angle-of-attack is made to track the outputs of a model of the same aircraft at low angle-of-attack, thereby emulating its good dynamics. An improved solution is the extension to combine the model-following controller with a controller that can augment the yaw control power by effectively modulating forebody pneumatic vortex flow nozzles. This paper presents the synthesis and design of both high angle-of-attack lateral/directional controllers for a generic X-29A equipped with forebody vortex flow control nozzles. Results demonstrate that both controllers are effective in improving closed-loop tracking of input commands and high angle-of-attack maneuverability, compared to a conventional flight control scheme using only aerodynamic control effectors, and without model-following.

**Nomenclature**

<table>
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<td>H</td>
<td>altitude</td>
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<td>h</td>
<td>integration stepsize</td>
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<td>M</td>
<td>Mach number</td>
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<tr>
<td>m</td>
<td>number of continuous control variables</td>
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<td>n</td>
<td>number of state variables</td>
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<td>p</td>
<td>number of bang-bang variables, also perturbed body axis roll rate</td>
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<tr>
<td>psf</td>
<td>pounds per square feet</td>
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<td>$\bar{q}$</td>
<td>dynamic pressure</td>
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<td>r</td>
<td>perturbed body axis yaw rate</td>
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<td>V</td>
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<td>PVC</td>
<td>pneumatic vortex control</td>
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<tr>
<td>$\alpha$</td>
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<td>$\mathbb{R}$</td>
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**Superscripts**

- $^*$ star trajectory
- $\cdot$ time derivative
- $'$ transpose of matrix or vector
- $^-1$ inverse of square matrix

**Subscripts**

- $\text{bb}$ bang-bang effector
- $c$ canard
- $\text{com}$ commanded value
- $\text{df}$ differential flap
- $k$ value at $k^{th}$ sample time
- $m$ model aircraft
- $r$ rudder
- $\text{sf}$ symmetric flap
- $\text{stf}$ strake flap
- $\text{tr}$ tracker aircraft
- $l$ trim value

**Introduction**

The deployment of lethal, reliable, all-aspect, short range missiles such as the AIM-9X series into the modern air combat arena has diminished the emphasis on sustained maneuvering capabilities [1]. These "point-and-shoot" weapons have prompted interest in controlled flight at angles-of-attack well beyond that for maximum lift.
Post-stall maneuvering in the low-speed, high angle-of-attack portion of the flight envelope, popularly referred to as agility, is motivated by this weapon technology [2]. Lan indicates in [3] that the agility of fighter type aircraft at high angles-of-attack can be seriously degraded because of the following factors:

1. unstable pitch break
2. ineffective propulsion/airframe integration
3. loss of static and dynamic lateral-directional stability
4. reduced control effectiveness

Items 3 and 4 above are of primary concern in this paper. Roll and yaw control effectiveness of fighter configurations typically decreases with increasing angle of attack due to separated flow and spanwise flow on swept wings, asymmetric vortex shedding, or adverse interference from stalled wing flow. The effects can be somewhat countered and lateral agility improved by carefully "tuning" the distribution of available control powers and their rates [4].

Tangential slot blowing using forebody Pneumatic Vortex Nozzles (PVC), a type of bang-bang control effector, for controlling the forebody vortex on fighter type aircraft at high angles of attack (Figure 1) is generally well understood [5, 6]. A version of the X-29A aircraft fitted with compressed nitrogen gas fed PVC nozzles mounted on each side of the forebody, was used to study methods to reduce the loss of directional control power at high angles of attack [7].

As tested in flight, this system proved extremely effective for augmenting directional control power at high angles-of-attack. However, the PVC nozzles were controlled manually by the pilot. During one test flight an earlier than planned manual initiation of the left PVC nozzle caused the X-29A to depart from controlled flight to the left at the start of a planned right roll maneuver [8]. Adams et al. in [9] synthesized a closed-loop PVC controller for the VISTA F-16 by designing a switching surface and implementing a deadband. Another approach is to use the methodology of Valasek in [10, 11, 12], which is to synthesize a Model Predictive Variable Structure Controller to properly modulate the PVC nozzles at high angle-of-attack, as successfully demonstrated for a generic X-29A type aircraft in [13]. Valasek further extended those results by implementing an optimal setpoint controller to command changes in aircraft attitude [14]. This system was limited to constant commands. Linse had previously in [15] extended the optimal setpoint controller to track time varying commands using the Command Generator Tracker (CGT) [16] for the same aircraft at high angle-of-attack. However, this aircraft did not have PVC nozzles.

This paper extends all previous work by using a variation of the CGT called the Model-Following Command Generator Tracker [17]. Its purpose is to drive an aircraft with undesirable dynamics to track the time varying outputs of an aircraft dynamic model with desirable dynamics. This controller is then integrated with the Model Predictive Variable Structure Controller (described in the next section) to permit use of PVC nozzles. The result is a controller which provides both good dynamic characteristics and maneuverability at high angles-of-attack.

**Systems With Mixed Continuous and Bang-Bang Control Effectors**

Figure 2 shows a block diagram for a generic system of this type. In [10] a design methodology is developed for directly designing variable structure controllers for this type of system. This methodology makes it possible to tradeoff system response, continuous effector activity, and bang-bang effector activity.
Consider a controllable and observable sampled-data control system consisting of a continuous plant, and a discrete controller which uses continuous effectors and bang-bang effectors, e.g. forebody vortex nozzles, as controls. For the linear time-invariant (LTI) case the state difference equation is

\[ X_{k+1} = \Phi(h)X_k + \Gamma(h)U_k + \Gamma_{bb}(h)U_{bb,k} \]  

where \( \Phi \) is an \( n \) by \( n \) discrete state transition matrix. The discrete control effectiveness matrices \( \Gamma \) and \( \Gamma_{bb} \) are functions of the integration stepsize, \( h \), and correspond to the continuous and bang-bang effectors respectively. The state vector is \( X_k \in \mathbb{R}^n \), and the control vector containing only continuous effectors is \( U_k \in \mathbb{R}^m \). The control vector containing only bang-bang effectors is \( U_{bb} \in \mathbb{R}^p \), such that:

\[ U_{bb} = \begin{cases} 1 \\ 0 \\ -1 \end{cases} \]  

A controller for the continuous effectors can be synthesized using the sampled-data regulator (SDR) and its associated weighting matrices \([18]\). The SDR cost function to be minimized is

\[ J = \frac{1}{2} \sum_{k=0}^{\infty} \left( X_k' \hat{Q} X_k + U_k' \hat{R} U_k + 2 X_k' M U_k \right) \]  

where \( \hat{Q}_k \in \mathbb{R}^{\text{con}} \) positive semi-definite state weighting matrix, \( \hat{R}_k \in \mathbb{R}^{\text{con}} \) positive definite control weighting matrix, and \( M \in \mathbb{R}^{\text{con}} \) positive semi-definite matrix which weighs the product of states and controls. This cost function and its weighting matrices are used to obtain the optimal constant feedback gains

\[ K = \left( \hat{R} + \Gamma' \Gamma \right)^{-1} \left( \Gamma' \Phi \Gamma + M \right) \]  

where \( P \) is the Ricatti matrix. Note that the SDR is not required to design the feedback gains in the present method; any technique can be used provided the gains are stabilizing. The benefit of using the SDR technique is the systematic progression from one candidate controller design to another. The feedback loop is closed with the control law

\[ U_k = -KX_k \]  

which accounts for the continuous effectors only. The closed-loop difference equation then has the form

\[ X_{k+1} = \left( \Phi(h) - \Gamma(h)K \right) X_k + \Gamma_{bb}(h)U_{bb,k} \]  

with the vector \( U_{bb} \) given by equation (2).

To control the bang-bang effectors, it is noted that at every sample interval, the discrete controller can conceivably choose between one of three mutually exclusive control strategies:

1. continuous effector active, bang-bang effector active in the positive position
2. continuous effector active, bang-bang effector inactive
3. continuous effector active, bang-bang effector active in the negative position

Strategies one and three above represent a continuous effector with a bang-bang effector included for an extra increment of control power. Normally this occurs for systems or in applications where the bang-bang effector possesses equal or greater control power than the continuous effector. Another consideration is that the bang-bang effector might be a finite resource to be conserved as much as possible. The pneumatic jets on the X-29A which are fueled with stored gas are an example.

**Bang-Bang Switching Logic**

The switching times for the bang-bang effector are determined by generating future system trajectories before the control is updated, using on-line simulation with the bang-bang effector in each of the (+), (0), and (-) states (Figure 3). The candidate structure which generates the lowest cost (state deviation and control activity) in terms of a performance index

\[ J_k = X_k' \hat{Q} X_k + U_k' \hat{R} U_k + 2 X_k' M U_k \]  

whose weighting matrices are selected by the designer, is selected and used during the upcoming sample interval. The operation of the continuous effector is unaffected. The on-line simulation can use either a full set of nonlinear dynamic equations or a reduced-order linear approximation. In the generic X-29A design examples which follow, a full set of linearized lateral/directional equations of motion are used.
Stability and robustness properties of this variable structure controller are presented in [12]. Robustness investigations demonstrated that the closed-loop PVC controller is reasonably robust (maximum 15.5% difference in cost) with respect to large parameter variations in yawing moment due to sideslip angle \( N_\beta \), even without sideslip angle feedback.

**Model-Following Command Generator Tracker**

In its basic form, the CGT commands system outputs to track time varying dynamic model outputs. In the simplest case, the model dynamics are stationary, resulting in the Nonzero Setpoint (NZSP) structure. It is capable of tracking only constant command inputs. By using a desired set of non-stationary model dynamics, time varying model outputs can be tracked with a feedforward structure. Most importantly, and of significance to this research, the tracking dynamics not only have zero steady-state error, but also force the system dynamics to emulate the model dynamics [17]. The major restrictions are that both the model and tracker dynamics must be observable and controllable. Also, the maximum number of model outputs that can be tracked is equal to the number of available control inputs.

Since the objective of the CGT controller is to improve the dynamic characteristic of an aircraft, the aircraft with poor dynamic characteristic is chosen as the tracker and the aircraft with desirable dynamic characteristic is selected as the model. The selections of the tracker and the model are unrestricted; it can be either the same aircraft at different flight conditions or two different aircraft.

The CGT of Reference 17 is derived as follows. The discrete command model to be followed by the tracker model is expressed in matrix form as

\[
\begin{bmatrix}
X_{m,k+1} \\
Y_{m,k}
\end{bmatrix} =
\begin{bmatrix}
\Phi_m & \Gamma_m & X_{m,k} \\
H_m & D_m & U_{m,k}
\end{bmatrix}
\]  

While the tracker output, \( y_k \), is tracking the model outputs, \( y_{m,k} \), the system states and controls are following some trajectory. The main objective of the CGT problem is to find an optimal state and control trajectory in steady-state during tracking known as the optimal star state and control, \( X^*_k, U^*_k \), so that their output produces an ideal star trajectory, \( y^*_k \), which satisfies

\[
y^*_k = y_m
\]

Then (8) can be expressed as

\[
\begin{bmatrix}
X_{m,k+1} \\
Y_{k}
\end{bmatrix} =
\begin{bmatrix}
\Phi_m, X_{m,k} + \Gamma_m U_{m,k} \\
H_m, X_{m,k} + D_m U_{m,k}
\end{bmatrix}
\]

Expanding this equation gives

\[
\begin{bmatrix}
X_{m,k+1} \\
Y_{k}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
X_{m,k} \\
U_{m,k}
\end{bmatrix}
\]

To develop the CGT, it is assumed that the star trajectory is linearly related to the model states and controls by

\[
\begin{bmatrix}
X^*_k \\
U^*_k
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
X_{m,k} \\
U_{m,k}
\end{bmatrix}
\]

The sub-matrices, \( A_{ij} \), will provide the mapping of the model onto the star trajectory. Incrementing the \( X^*_k, X_{m,k} \) relationship in (12) gives

\[
\begin{bmatrix}
X^*_{k+1} - X^*_k \\
U^*_{k+1} - U^*_k
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
X_{m,k+1} - X_{m,k} \\
U_{m,k+1} - U_{m,k}
\end{bmatrix}
\]

Expanding this equation provides

\[
\begin{bmatrix}
X^*_{k+1} - X^*_k \\
U^*_{k+1} - U^*_k
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
X_{m,k+1} - X_{m,k} + A_{12} U_{m,k+1} - U_{m,k} \\
X_{m,k+1} - X_{m,k} + A_{22} U_{m,k+1} - U_{m,k}
\end{bmatrix}
\]  

Combining the first row of (14) and the second row of (11) gives
\[
\begin{bmatrix}
X_{k+1}^* - X_k^* \\
\dot{y}_k^*
\end{bmatrix} =
A_{11} \begin{bmatrix}
X_{m,k+1} - X_{m,k} \\
H_m X_{m,k} + D_m \dot{U}_m
\end{bmatrix} + A_{12} \begin{bmatrix}
\dot{X}_{m,k} \\
\dot{U}_m
\end{bmatrix}
\]  

Further, assume that the controls of the model are constant
\[
U_{m,k+1} - U_{m,k} = 0
\]  

After this assumption, (15) becomes
\[
\begin{bmatrix}
X_{k+1}^* - X_k^* \\
\dot{y}_k^*
\end{bmatrix} =
A_{11} \begin{bmatrix}
X_{m,k+1} - X_{m,k} \\
H_m X_{m,k} + D_m \dot{U}_m
\end{bmatrix}
\]  

where the index on \(U_m\) has been dropped because it is constant. Substituting \(X_{m,k+1}\) of (11) into (17) provides
\[
\begin{bmatrix}
X_{k+1}^* - X_k^* \\
\dot{y}_k^*
\end{bmatrix} =
A_{11} \begin{bmatrix}
\Phi_m - I \\
H_m + D_m \dot{U}_m
\end{bmatrix}
\]  

The original star trajectory state and output equations must satisfy
\[
\begin{bmatrix}
X_{k+1} \\
\dot{y}_k
\end{bmatrix} =
\begin{bmatrix}
\Phi & \Gamma \\
H & D
\end{bmatrix}\begin{bmatrix}
X_k \\
\dot{U}_k
\end{bmatrix}
\]  

Subtracting \(X_k^*\) from both sides of the state equation gives the increment star trajectory relations
\[
\begin{bmatrix}
X_{k+1}^* - X_k^* \\
\dot{y}_k^*
\end{bmatrix} =
\begin{bmatrix}
\Phi - I & \Gamma \\
H & D
\end{bmatrix}\begin{bmatrix}
X_k^* \\
\dot{U}_k^*
\end{bmatrix}
\]  

Then substituting (12) into (20) provides the increment star trajectory in terms of state and control of the command model
\[
\begin{bmatrix}
X_{k+1}^* - X_k^* \\
\dot{y}_k^*
\end{bmatrix} =
\begin{bmatrix}
\Phi - I & \Gamma \\
H & D
\end{bmatrix}\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}\begin{bmatrix}
X_{m,k} \\
\dot{U}_m
\end{bmatrix}
\]  

The left hand side of (18) and (21) are identical. Therefore, equating the right hand sides of the two equations give
\[
\begin{bmatrix}
\Phi - I & \Gamma \\
H & D
\end{bmatrix}\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} =
\begin{bmatrix}
A_{11}(\Phi_m - I) & A_{11} \Gamma_m \\
H_m & D_m
\end{bmatrix}
\]  

The matrix on the far left is the quad partition matrix (QPM). Assuming the QPM is invertible, its inverse is defined as
\[
\begin{bmatrix}
\Phi - I & \Gamma \\
H & D
\end{bmatrix} =
\begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{21} & \Pi_{22}
\end{bmatrix}
\]  

so that the sub-matrices, \(A_{ij}\), can be solved for
\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} =
\begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{21} & \Pi_{22}
\end{bmatrix}\begin{bmatrix}
A_{11}(\Phi_m - I) & A_{11} \Gamma_m \\
H_m & D_m
\end{bmatrix}
\]

Separating each entity,
\[
\begin{align*}
A_{11} &= \Pi_{11} A_{11}(\Phi_m - I) + \Pi_{12} H_m \\
A_{12} &= \Pi_{11} A_{11} \Gamma_m + \Pi_{12} D_m \\
A_{21} &= \Pi_{11} A_{11}(\Phi_m - I) + \Pi_{22} H_m \\
A_{22} &= \Pi_{11} A_{11} \Gamma_m + \Pi_{22} D_m
\end{align*}
\]

The first equation is a Lyapunov equation that is a function of \(A_{11}\) only. It has a solution if
\[
\lambda(\Pi_{11}) \lambda(\Phi_m - I) \neq 1
\]

Once \(A_{11}\) is found using the Lyapunov equation, the other equations for \(A_{12}, A_{21}\) and \(A_{22}\) are simple evaluations. The feedback control law with CGT feedforward terms is
\[
U_k = -K \dot{X}_k + (\Pi_{21} + K \Pi_{11}) \dot{X}_{m,k} + (\Pi_{22} + K \Pi_{12}) \dot{U}_m
\]

Note that the nonzero setpoint (NZSP) controller is just a special case of the CGT without model dynamics:
\[
\begin{align*}
\Phi_m &= I \\
\Gamma_m &= 0 \\
H_m &= 0 \\
D_m &= I
\end{align*}
\]

**Simulation Models**

All models are of a generic X-29A. The flight condition of interest for the tracker is very low speed, high angle-of-attack, where the dynamics are slow and damping generally deficient.

\[
\begin{align*}
M_{1,n} &= 0.35 \\
M_{1,r} &= 38000. \text{ feet} \\
\alpha_{1,r} &= 40^\circ \\
V_{1,r} &= 338 \text{ feet/sec} \\
\delta_{u} &= 37 \text{ psf} \\
\delta_{\alpha} &= -24.4^\circ \\
\delta_{\delta_{u}} &= 12.8^\circ \\
\delta_{\delta_{\alpha}} &= 20.7^\circ
\end{align*}
\]

Control power of the PVC nozzles is comparable to that of...
The generic X-29A linear tracker model corresponding to this flight condition is obtained from Reference 13:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{p} \\
\dot{r} \\
\dot{\phi}
\end{bmatrix} = 
\begin{bmatrix}
0.43 & 0.64 & -0.77 & 0.073 \\
-8.39 & 0.78 & -0.69 & -0.74E-5 \\
0.14 & 0.063 & -0.11 & -0.17E-5 \\
0. & 1. & 0.84 & 0.
\end{bmatrix}
\begin{bmatrix}
\beta \\
p \\
r \\
\phi
\end{bmatrix}
+ 
\begin{bmatrix}
-0.034 \\
0.86 \\
0.92 \\
0.0
\end{bmatrix}
\begin{bmatrix}
\delta_{df} \\
\delta_{dr} \\
\delta_{fr} \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
0.013 \\
0.12 \\
0.0 \\
0.0
\end{bmatrix}
\delta_{fyn}
\]

where all angular quantities are in radians. The X-29A tracker has three lateral/directional control effectors: two continuous (differential flaps and rudder), and one bang-bang (PVC nozzles). The open-loop eigenvalues are \(\lambda_{1,2} = 0.42 \pm 2.32j\), \(\lambda_{3,4} = -0.069 \pm 0.12j\), corresponding to an unstable Dutch roll mode with frequency and damping ratio \(\omega_{DR} = 2.36\) rad/sec, \(\zeta_{DR} = -0.18\). A lateral phugoid mode with frequency \(\omega = 0.14\) rad/sec, and damping ratio \(\zeta = 0.49\) replaces the roll mode and spiral mode.

The simulation model used in the examples is equations (29), and includes reduced order actuator dynamics of the real X-29A actuators in [19]. A conservative time constant of 0.125 seconds for the PVC nozzles produces a variation in total cost of only 4% compared to a no-lag nozzle, and is used in the examples.

**Controller Design**

The objective is to design a closed-loop lateral/directional PVC controller that will make the high angle-of-attack system (the tracker) follow the trajectory of the low angle-of-attack system (the model). The states are two independent controls, two commanded outputs are available. In this study, sideslip angle and bank angle of the model are selected as the trajectory parameters to be tracked by the tracker. Both sideslip and bank angles of the model aircraft are commanded to perform certain maneuvers that are specified by the designer using the NZSP structure. Then, the tracker aircraft traces both sideslip and bank angles of the model aircraft trajectories by employing the CGT controller. It is important to note that the entire NZSP structure is simply an underlying process to convert the step input commands to the input to the CGT controller which is the desirable trajectories of the model aircraft. All states are assumed to be fed back (including sideslip angle) and perfectly measurable. The fuel supply of the PVC nozzles is considered to be finite, so a controller which satisfies the specifications with the least amount of bang-bang activity is desired.

The lateral/directional sampled-data regulator (SDR) controller uses a 10 Hz sampling rate, which is 20 times faster than the fastest frequency in the lateral/directional model. Initial elements of the state and control SDR weighting matrices are selected using the relations in [20]. Using systematic iterations the following weights were selected to generate a nominal SDR controller design:

\[
Q_{11} = 10000 \quad Q_{22} = 9000
\]
\[ Q_{33} = 2000 \quad Q_{44} = 365 \]
\[ R_{11} = 85. \quad R_{22} = 2.5 \]

These weights result in tracker closed-loop eigenvalues \( \lambda_{1,2} = -0.39 \pm 2.2j \), \( \lambda_{3,4} = -0.50 \pm 0.48j \). The closed-loop Dutch roll has \( \omega_{DR} = 2.3 \text{ rad/sec} \), \( \zeta_{DR} = 0.17 \), and the closed-loop lateral phugoid has \( \omega = 0.70 \text{ rad/sec} \), and damping ratio \( \zeta = 0.72 \). For clarity the weights and SDR gains are held fixed for each candidate controller design. The only parameter varied is \( R_{33} \), the weight on the PVC nozzles. The nominal controller (no PVC) uses the MPVSC structure, but a sufficiently large initial value of \( R_{33} \) ensures that the PVC nozzles are never activated, as desired. A value of \( R_{33} = 10 \) serves this purpose, and the resulting nominal controller is labeled Design 1. Successive designs will be generated in the examples simply by selecting different weights on \( R_{33} \).

**Examples**

For simplicity, a set of commands was determined beforehand off-line using an NZSP result. The first example is intended to show the good tracking properties of the CGT by reversing the roles of tracker and model (as previously defined), and Design 1 is used as the controller. The high angle-of-attack model is commanded to perform two consecutive turns, the first at time = 1 second (\( \beta = -1 \text{ degree} \), \( \phi = -5 \text{ degrees} \)), and the second at time = 10 seconds (\( \beta = 1 \text{ degree} \), \( \phi = 5 \text{ degrees} \)). The high speed, low angle-of-attack tracker follows its output. Figure 4 shows that the low angle-of-attack tracker (fast dynamics) is being driven so as to exhibit the poorly damped dynamics of the high angle-of-attack model (slow dynamics), as desired. Tracking of the sideslip angle and bank angle outputs is excellent, with the only tracking errors occurring where the model outputs are changing rapidly, as expected. The differential flap actuator saturation that occurs at 12 seconds is of the model actuator, not the tracker actuator. Therefore, there is no "real" actuator being saturated.

Reverting back to the original problem as posed, Design 2 is now used to make the high angle-of-attack tracker exhibit the desirable dynamics of the low angle-of-attack model. The same command inputs are used. The results displayed in Figure 5 show that as desired, the high angle-of-attack tracker now exhibits the well damped dynamics of the low angle-of-attack model. This is possible because of the generally good tracking of the model outputs. Most significantly for a high angle-of-attack flight condition, note that neither the differential flap nor rudder actuators saturate. Naturally, since the tracker dynamics are slower than the model dynamics, tracking performance is not ideal, and can degrade significantly if the model outputs vary too rapidly.

The intent of the next example is to demonstrate the effectiveness of the CGT implementation with the MPVSC controller. This is done by using more aggressive command inputs, thereby requiring use of the PVC nozzles. The low angle-of-attack model is commanded from initial to terminal sideslip angles of 4 to -1 degrees, and bank angles from 10 to -10 degrees. This initial condition is sufficiently large enough at this flight condition to excite the Dutch roll and saturate the differential flaps and rudder, but prevent the aircraft from departing controlled flight [14]. Design 3 shows the response to the new command inputs using the same controller used in Design 1 in Figure 6 (PVC nozzles not activated). The response is lightly damped in both sideslip and bank angles, with the differential flap and rudder on their rate and position limits for the entire 10 second duration. The total cost using equation (7) is 2540.

The weight on the forebody nozzles only is now reduced to 0.9 to generate Design 4. Time history responses for the same commands are displayed in Figure 7. Design 4 has a cost of 1018 compared to the 2540 of Design 3, a reduction of 60%. Proper modulation of the forebody nozzles by the MPVSC controller to supply the additional required yaw control power results in much improved sideslip angle and bank angle responses, as steady-state for both is achieved within the first six seconds. The differential flap and rudder activities are greatly reduced, with the differential flap saturating twice, and the rudder not at all. However, wear and tear on the differential flap and rudder are being traded for wear and tear on the valves of the forebody nozzles. This is a design tradeoff.

**Summary and Conclusions**

Two different controller structures were synthesized and demonstrated for high angle-of-attack lateral/directional flight control of a generic X-29A. The first used a Model-Following Command Generator Tracker to drive the outputs of an aircraft at a high angle-of-attack flight condition to effectively track the time varying outputs of a dynamic model of the same aircraft at a low angle-of-attack flight condition. The second controller is an extension of the first, employing forebody vortex flow control effectors, integrated with a Model-Following Command Generator Tracker to a Model Predictive Variable Structure Controller. This provides the previously mentioned benefits, together with effective control of forebody vortices to enhance high angle-of attack maneuverability. The closed-loop control laws are full
state feedback, and are synthesized using a linear time-invariant generic model of the X-29A. Based upon the results of this work it is concluded that:

1. For the aircraft type, flight condition, and command inputs shown, results show that the Model-Following Command Generator Tracker can permit an aircraft flying at a low speed, high angle-of-attack flight condition (with undesirable dynamics) to effectively track the time varying outputs of a high speed, low angle-of-attack aircraft model (with desirable dynamics), using only aerodynamic control effectors.

2. Proper integration of the Model-Following Command Generator Tracker to an existing Model Predictive Variable Structure Controller can provide significant improvements in high angle-of-attack maneuvering response characteristics. This is a direct result of the improved tracking of time varying model outputs, combined with the proper utilization of forebody vortex nozzles.

3. The methodology demonstrated permits the designer to select the activity level of the forebody vortex nozzles to achieve good closed-loop tracking performance. This is done by directly trading-off use of the forebody vortex nozzles and the conventional aerodynamic effectors.

Acknowledgment
This research was sponsored by the Texas A&M University Program to Enhance Scholarly and Creative Activities, and by AFOSR Grant F49620-93-C-0063. The project monitors for the AFOSR Grant were Dr. Lawrence A. Walchli and Captain Brian A. Parker. All of this support and assistance is gratefully appreciated by the authors.

References


Figure 4 Design 1, $R_{33}=10$, Generic X-29A, High $\alpha$ Model, Low $\alpha$ Tracker

Figure 5 Design 2, $R_{33}=10$, Generic X-29A, Low $\alpha$ Model, High $\alpha$ Tracker
Figure 6 Design 3, $R_{33} =$10, Tracker Cost = 2540
Figure 7  Design 4, $R_{33} = 0.9$, Tracker Cost = 1018
Generic X-29A, Low $\alpha$ Model, High $\alpha$ Tracker