System Identification of an Unmanned Aerial Vehicle with Hingeless Control Effectors

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This paper presents research conducted for system identification of an unmanned aerial vehicle to quantitatively compare control authority of conventional ailerons and jet actuators in lateral/directional motion. Locally linear state-space models were identified, via the Observer/Kalman filter algorithm, to reveal the dynamics of the unmanned aerial vehicle test-bed for different roll control effector configurations. The accuracy of the generated models is demonstrated by examination of residual error between the recorded flight states and the identified state-space model output using the recorded pilot inputs. Linear systems theory is used to (1) identify correct linearized model order for minimal realization of the equations of motion, (2) verify consistent plant dynamics across roll effector configurations, and (3) quantitatively assess roll effector authority across the configurations. The identified system models yield a quantitative comparison of control effectiveness of conventional and proposed aileron actuators which align with pilot feedback and test observations.

Nomenclature

\( \mathbf{X}, \mathbf{Y}, \mathbf{Z} \) Reference Coordinate System Axes: Primary, Secondary, and Tertiary
\( \Phi, \Theta, \Psi \) Euler Angles: Bank, Elevation, and Heading
\( \dot{\Phi}, \dot{\Theta}, \dot{\Psi} \) Euler Angular Rates
\( \mathbf{x}, \mathbf{y}, \mathbf{z} \) Vehicle Coordinate System Axes: Primary, Secondary, and Tertiary
\( u, v, w \) Vehicle Coordinate System Translational Rates
\( \phi, \theta, \psi \) Vehicle Coordinate System Angular Displacements: Roll, Pitch, and Yaw
\( \mathbf{\omega} \) Vehicle Angular Velocity Vector in Vehicle Frame
\( p, q, r \) Vehicle Coordinate System Angular Rates
\( \mathbf{F} \) Force Vector
\( \mathbf{V} \) Velocity Vector
\( \mathbf{W} \) Weight Vector
\( m \) Vehicle Mass
\( \mathbf{M} \) Moment Vector
\( \mathbf{I} \) Inertia Tensor
\( \mathbf{V}_T, \alpha, \beta \) Stability Coordinate System: Total Velocity, Angle-of-Attack, Sideslip Angle
\( \delta \) Control Effector Input

Superscript
\( \mathbf{V} \) Vehicle Frame
\( \mathbf{N} \) Reference/Inertial Frame

Subscript
\( \text{latd} \) Lateral-Directional Motion
\( \text{long} \) Longitudinal Motion

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I. Introduction

The ability to construct accurate, locally linear aircraft system models is becoming more important in autonomous and intelligent flight control applications where primary control decisions are shifted to an onboard system with humans in an outer, secondary, control loop. Since primary control is handed to an onboard system, identification of the plant model and control effector authority are extremely important for retention of stable vehicle operation. Salman et al.\textsuperscript{1} used a nonlinear mapping method to produce nonlinear state-space models from collected flight data composed of only a trim flight state without any control effector input. Another approach for control effector identification is to use Global-Local Mapping\textsuperscript{2} composed of a locally linear model and a nonlinear model and has been applied to form a composite vehicle system model.\textsuperscript{3} However, a linear model must still first be identified before the nonlinear model can be applied to form the composite solution.

For linear system identification, many methods are available for expressing a nonlinear system as a linearized system about a nominal or trim state using either frequency or temporal methods. The three most common methods are fast Fourier transforms, maximum likelihood estimation, and least-squares. Ho and Kalman\textsuperscript{4} introduced an alternative approach utilizing the concept of minimal realization to produce the smallest dimension state-space model that was able to effectively recreate the input-output system character. This minimal realization was built on by Juang and Pappa producing the Eigensystem Realization Algorithm (ERA). ERA is a time domain technique seeking to identify system Markov parameters from input-output data with original applications to flexible structures and systems with sensor noise.\textsuperscript{5, 6} The limiting conditions of the ERA approach were that the initial conditions and control had to be zero and the perturbed system response had to decay to a zero steady-state value. For computation the input matrix had to be invertible so lightly damped structures necessitated large storage due to excessive lengths of input-output time history data. For flight vehicle application, the large storage requirement and processing time do not allow for near real-time computation and the need for zero initial conditions is contrary to typical aircraft flight trim states.

The Observer/Kalman filter Identification (OKID) technique\textsuperscript{7} was developed as an extension of ERA that could be used with correlated data using the Eigensystem Realization Algorithm with Data Correlation (ERA/DC) technique. OKID was developed for efficient identification of large flexible structures, particularly spacecraft, and is based upon stochastic estimation and deterministic Markov parameter identification techniques.\textsuperscript{8} The OKID algorithm produces a discrete, locally linear system state-space model and is able to accommodate nonzero initial conditions, significantly reducing the required data record and computation time by introduction of a deadbeat observer to compress the data and improve the identification results. OKID requires measurement of appropriate system states and input magnitudes to excite the system modes sufficiently without driving the system outside of a linear operating range. The advantages of the OKID algorithm for linear model identification allow for application in near real-time for aircraft system identification at different operating conditions. Valasek\textsuperscript{9} has shown application of OKID to simulated aircraft flight operations is possible without major modification and system models are able to be produced with different trim conditions, sensor noise, and discrete disturbances for simulated aircraft.

The objective of this study was to quantitatively compare lateral/directional control effectiveness for different aileron configurations on the same unmanned aircraft test-bed. Two configurations utilized conventional ailerons with one operating inboard and outboard ailerons while the second operated with only inboard ailerons. The third configuration replaces the outboard conventional ailerons with jet actuators using directed air jets to create the pressure difference needed for roll moment generation. Section II provides the nonlinear dynamic modeling equations for lateral/directional and longitudinal motion along with the derivation of the minimal linearized state-space models for decoupled motion, Section III provides the background theory for the Observer/Kalman filter Identification Algorithm, while Section IV gives the results of the OKID algorithm applied to the Extra 300 test-bed flight data for longitudinal and lateral/directional system models, and Section V provides the conclusions drawn from this study.
II. Aircraft Dynamic Modeling Equations

A. Kinematic Equations

For appropriate dynamic characterization of an aircraft in three dimensions, Newton’s Second Law is applied for force and moment resolution for derivation of equations of motion of a rigid body. Proper application of Newton’s Second Law requires the summation of external forces on a body be equal to the body’s time rate change of linear momentum and the summation of moments be equal to the body’s time rate change of angular momentum, as shown in Equations (1) and (2).

\[ \sum F_V + W_V = \frac{d}{dt} (mV) \]  \hspace{1cm} (1)

\[ \sum M_V = \frac{d}{dt} (I_V \vec{\omega}) \]  \hspace{1cm} (2)

Equations (1) and (2) define motion in a local coordinate system, i.e. the vehicle’s local frame. Vehicle frame is designated by using vehicle center of gravity as the coordinate frame origin with the primary vehicle axis, \( x \), extending positively out the front of the vehicle, the vehicle secondary axis, \( y \), extending positively out the starboard side of the vehicle, and the tertiary vehicle axis, \( z \), extending in the direction of the cross-product of the primary and secondary axes. Using this axis convention the Inertia tensor is defined by Equation (3).

\[ I_{veh} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \]  \hspace{1cm} (3)

Where:

\[ I_{ij} = \iiint m (ij) \, dm \]

\[ I_{ii} = \iiint m (j^2 + k^2) \, dm \]

Since Newton’s second law is only valid in inertial frame Equations (1) and (2) must be in a reference or inertial frame. To relate the motion measured in vehicle frame to a reference, fixed or inertial, frame the rate of change of an arbitrary vector measured in vehicle frame \( \vec{G}_V \) while the vehicle possesses an angular velocity, \( \vec{\omega} \), can be related to the reference frame by Equation (4)\(^{10}\) where the superscripts denote differentiation with respect to either inertial frame, \( \mathcal{N} \), or vehicle frame, \( \mathcal{V} \), derivatives.

\[ \frac{d\mathcal{N}}{dt} \vec{G}_V = \frac{d\mathcal{V}}{dt} \vec{G}_V + \vec{\omega} \times \vec{G}_V \]  \hspace{1cm} (4)

Equation (4) is applied to Equations (1) and (2) to relate the force and moment experienced in the vehicle frame to the inertial frame of reference resulting in Equations (5) and (6). Using the Flat-Earth assumption, the inertial frame origin is taken to be at ground level with the primary axis, \( X \), extending positively eastward, the tertiary axis, \( Z \), pointing positively in the direction of the local gravity vector, \( \vec{g} \), and the secondary axis, \( Y \), pointing in the direction of the cross-product of the tertiary and primary axes.

\[ \sum F_\mathcal{N} = \frac{d}{dt} (mV) + \vec{\omega} \times (mV) \]  \hspace{1cm} (5)

\[ \sum M_\mathcal{N} = \frac{d}{dt} (I_V \vec{\omega}) + \vec{\omega} \times (I_V \vec{\omega}) \]  \hspace{1cm} (6)

The scalar forms of Equations (5) and (6) for calculation of the net force and moment terms along the vehicle axes expressed in inertial frame are given below in Equations (7) and (8).

\[ \left\{ \begin{array}{c} F_X \\ F_Y \\ F_Z \end{array} \right\}_\mathcal{N} = W_\mathcal{N} + F_{Aerodynamic} + F_{Thrust} = \left\{ \begin{array}{c} m(\dot{u} - vr + wq) \\ m(\dot{v} + ur - wp) \\ m(\dot{w} - uq + vp) \end{array} \right\} \]  \hspace{1cm} (7)
Equation (7), vehicle force equation, accounts for vehicle weight, aerodynamic, and propulsive forces along the vehicle axes while Equation (8), vehicle moment equation, accounts for moments about the vehicle axes arising from these forces as well as vehicle rotation rates. The thrust force on the aircraft is assumed to be present along the primary vehicle axis for flight testing and simulations. This is a valid assumption since the Extra 300 is a propeller driven aircraft which is mounted approximately along the vehicle primary axis.

For determination of the vehicle orientation with respect to the reference coordinate system, a coordinate transform between vehicle and reference coordinate systems is required. This is achieved through Euler angle rotations about the reference coordinate system in a specific sequence whose order is important. This work uses the “aerospace” or “3-2-1” sequence for the inertial to vehicle transform derivation. This sequence is rotations about the reference coordinate system in a specific sequence whose order is important.

\[
\dot{\omega} \times \mathbf{I}_V \omega = \begin{cases} 
q r (I_{yy} - I_{zz}) + (q^2 - r^2) I_{yz} - pr I_{xy} + pq I_{xz} \\
p r (I_{zz} - I_{xx}) + (r^2 - p^2) I_{xz} - pq I_{yz} + qr I_{xy} \\
p q (I_{xx} - I_{yy}) + (p^2 - q^2) I_{yz} - qr I_{xy} + pr I_{xy}
\end{cases}
\]  
(8)

Equation (9) gives the transformation governed by a rotation about the reference tertiary axis, rotation about the new secondary axis, and finally uses the "aerospace" or "3-2-1" sequence for the inertial to vehicle transform derivation. This sequence is rotations about the reference coordinate system in a specific sequence whose order is important.

\[
\mathbf{G}_V = T_{N2V} \ast \mathbf{G}_N = \begin{bmatrix}
C\Theta C\Psi & C\Theta S\Psi & -S\Theta \\
S\Phi S\Theta C\Psi - C\Phi S\Psi & S\Phi S\Theta S\Psi + C\Phi C\Psi & S\Phi C\Theta \\
C\Phi S\Theta C\Psi + S\Phi S\Psi & C\Phi S\Theta S\Psi - S\Phi C\Psi & C\Phi C\Theta
\end{bmatrix} \ast \mathbf{G}_{ref}
\]  
(9)

Note: \( C\phi = \cos(\phi) \) and \( S\phi = \sin(\phi) \)

The transformation matrix in Equation (9) is known as the Direction Cosine Matrix or DCM. Due to the nature of the inverse of orthonormal matrices, the transformation matrix from vehicle frame to inertial frame, \( T_{V2N} \), is the transpose of the matrix in Equation (9). Transference of vehicle translational vectors to inertial vectors utilizes the transpose of the Direction Cosine Matrix along with the addition of initial vehicle frame origin offset conditions, e.g. wind velocity or translation offset. Using the DCM, we can then transform the weight vector in reference frame, \( \mathbf{W}_N \), to vehicle frame in the following way:

\[
\mathbf{W}_V = T_{N2V} \ast \mathbf{W}_N = T_{N2V} \begin{bmatrix}
0 \\
0 \\
-mg
\end{bmatrix}
\]  
(10)

The transformation between Euler angular rates, reference frame angular rates, and vehicle rates is given by Equation (12). The vehicle angular rate vector is derived by replacing \( \mathbf{G}_V \) in Equation (4) with Equation (9) to yield Equation (11).

\[
[\dot{\omega} \times] = \frac{d}{dt} (T_{N2V}) (T_{N2V})^T
\]

Application of Equation (11) produces Equations (12) and (13) for transference of Euler rates to vehicle frame rates or vice-versa. Note that the inverse of the transformation matrix in Equation (12) must be used...
to solve for transferring vehicle angular rates to inertial angular rates instead of the transpose operation because the transformation matrix is not orthonormal.

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}_v = \begin{bmatrix}
1 & 0 & -\sin \Theta \\
0 & \cos \Phi & \cos \Theta \sin \Phi \\
0 & -\sin \Phi & \cos \Theta \cos \Phi
\end{bmatrix}_v \begin{bmatrix}
\dot{\Phi} \\
\dot{\Theta} \\
\dot{\Psi}
\end{bmatrix}_N
\] (12)

\[
\begin{bmatrix}
\dot{\Phi} \\
\dot{\Theta} \\
\dot{\Psi}
\end{bmatrix}_N = \begin{bmatrix}
1 & \sin \Phi \tan \Theta & \cos \Phi \tan \Theta \\
0 & \cos \Phi & -\sin \Phi \\
0 & \sin \Phi \sec \Theta & \cos \Phi \sec \Theta
\end{bmatrix}_N \begin{bmatrix}
p \\
q \\
r
\end{bmatrix}_v
\] (13)

Integration of either Equation (12) or (13) yields the total angular displacement over time for the coordinate system of interest. Equation (13) demonstrates the Gimbal Lock phenomena with a divide by zero error at \( \theta = \pm 90 \text{deg} \) where the bank and heading angles are unable to be computed due to the parallel alignment of the roll and pitch axes.

Equation (7), vehicle force equation, expressed vehicle velocity in terms of vehicle axes, but another coordinate system choice allows for a more direct analysis of vehicle stability behavior. Conversion from the vehicle to stability coordinate system allows for direct study of Dutch-roll, roll, and spiral modes of lateral/directional motion due to the simplification of linearized equations of motion.\textsuperscript{11} For longitudinal motion, the short period and phugoid modes can be described easily by either vehicle or stability axes. This study utilizes the stability coordinate system for both types of motion. The Stability coordinate system is preferred in many stability studies\textsuperscript{12} and is defined with the primary axis initially aligned with the relative wind vector while at a steady-state, trim, condition then rotates due to a disturbance or control input. A conventional aircraft at trim conditions, such as the Extra 300 test-bed, will align itself directionally with the relative wind vector due to the vertical tail directional authority resulting in “weathercock” stability.

It is convenient to use stability axes which are defined relative to the velocity vector, \( V_T \), after a disturbance or control input by two angles: angle-of-attack, \( \alpha \), and sideslip angle, \( \beta \). From the characteristic analysis of lateral/directional modes given by Nelson\textsuperscript{13}, Dutch-roll and spiral modes are primarily described by sideslip and yaw rate, while roll and spiral modes are primarily described by roll rate and roll angle. The kinematic equations describing the differential equations of angle-of-attack, sideslip, and wind velocity are given by Equations (14)-(16). Parameter transformations from stability axes to vehicle axes and vice versa are given by Equation (17).

\[
\dot{\alpha} = q - (p \cos \alpha + r \sin \alpha) \tan \beta - \frac{LOM}{V_T \cos \beta}
+ \frac{g}{V_T \cos \beta} (\cos \theta \cos \phi \cos \alpha + \sin \theta \sin \alpha) \tag{14}
\]

\[
\dot{\beta} = p \sin \alpha - r \cos \alpha + \frac{1}{V_T} (YOM \cos \beta + DOM \sin \beta)
+ \frac{g}{V_T} (\cos \theta \sin \phi \cos \beta + \sin \theta \sin \beta \cos \alpha - \cos \theta \cos \phi \sin \beta \sin \alpha) \tag{15}
\]

\[
\dot{V}_T = YOM \sin \beta - DOM \cos \beta
+ g \left[ (\cos \theta \cos \phi \sin \alpha - \sin \theta \cos \alpha) \cos \beta + \cos \theta \sin \phi \sin \beta \right] \tag{16}
\]

Where net drag, lift, and side forces are normalized with respect to vehicle mass, \( m \):

\[
DOM = \frac{D - T \cos \alpha}{m}, \quad YOM = \frac{Y}{m}, \quad LOM = \frac{L + T \sin \alpha}{m}
\]
Stability to Vehicle Coordinate Transform
\[
\begin{align*}
  u &= V_T \cos \alpha \cos \beta \\
  v &= V_T \sin \beta \\
  w &= V_T \sin \alpha \cos \beta \\
  \alpha &= \tan^{-1} \frac{w}{u} \\
  \beta &= \sin^{-1} \frac{v}{V_T} \\
  V_T &= \sqrt{u^2 + v^2 + w^2}
\end{align*}
\] (17)

Vehicle to Stability Coordinate Transform

B. Kinematic Response to Control Effectors

The preceding section presents the necessary differential equations to describe vehicle motion: Equations (7) or (14)-(16) for translational velocities, Equation (8) for angular rates, and integration of Equation (12) or (13) for angular displacement. In this study vehicle displacement was neglected so the typical 12 governing equations of motion were reduced to 9.

From the kinematic equations, it is not entirely evident how control effectors such as ailerons, rudder, elevator, or thrust impact vehicle motion since their effect is felt indirectly via changes in forces experienced by the vehicle. Additionally, the degree of nonlinearity does not allow for ease in examining state sensitivities with regard to the changes other states or control variables. Therefore, a linearization of the equations of motion is performed for study of aircraft behavior with regard to a trim point\textsuperscript{14}.

1. Motion Decoupling Assumptions and State Identification

For control authority quantization of different control effectors operating in different flight modes, a direct and preferably linear relationship must be identified between system states and control input. Proceeding in this manner, assumptions are made about an aircraft’s operating regime and the aircraft itself since an aircraft not only possess three degrees-of-freedom in translation and three degrees-of-freedom in rotation but also numerous elastic degrees of freedom which can be hard to capture due to operating condition dependence\textsuperscript{13}. Two major assumptions are made in this study which are typical of flight stability analysis: (I) the aircraft is a rigid body, and (II) the aircraft experiences only small disturbances.

The small disturbance assumption models the vehicle as not experiencing wild or chaotic motion when disturbed from trim but rather “small” deviations about a steady flight condition. Large excursions, spinning or stall, are not analyzed due to the relatively high degree of nonlinearity of the equations of motion. The rigid body assumption was made to make use of the Newton-Euler formulation for aircraft kinematics. These assumptions allow for the aircraft motion to be decoupled into two types: longitudinal and lateral/directional.

Longitudinal motion is confined to the plane formed by the vehicle primary and tertiary axes so only the primary axis and tertiary axis force equations along with pitching moment equations are necessary for motion description. Lateral/Directional motion is composed of the secondary axis force equation along with the rolling and yawing moment equations. Therefore \( u, w, q, \) and \( \theta \) can be used for longitudinal states and \( v, p, r, \phi, \) and \( \psi \) for lateral/directional states.

As mentioned earlier the vehicle coordinate system does not always provide the needed insight for dynamic behavior so the stability coordinate system is utilized by replacing \( u \) and \( w \) with \( V_T \) and \( \alpha \) for longitudinal motion and \( v \) with \( \beta \) for lateral/directional motion. The equations of motion for minimal state resolution now become Equations (8), (14)-(16), and the vehicle roll, pitch, and yaw angular displacements result from integration of their respective rates.

The Extra 300 test-bed is a conventionally controlled aircraft composed of inboard and outboard ailerons, rudder on a single vertical stabilizer, and elevator control surfaces along with a propeller driven engine for thrust production. Longitudinal motion control effectors are characterized by \( \delta_e \) and \( \delta_T \) for elevator deflection and thrust inputs. Lateral/Directional motion control is achieved through \( \delta_a \) and \( \delta_r \) characterizing aileron and rudder deflection for control input.
The nonlinear equations of motion for the stability coordinate system, \( f_i \), result in a 9-element state vector and 4-element control vector to characterize total aircraft motion shown by Equation (18).

\[
\dot{x}_i (t) = f_i \left( x (t), \dot{x} (t), u (t) \right) \quad i \in [1, 9] \\
x = [V_T, \alpha, q, \theta, \beta, p, r, \phi, \psi]^T \\
u = [\delta_T, \delta_c, \delta_a, \delta_r]^T
\]  

(18)

Control effectiveness is not directly observable through the equations of motion but is manifested through the change in forces and moments experienced by the aircraft as shown in Nelson\(^{11} \). Small-Disturbance Theory was applied about the aircraft at a trim state, designated with subscript “0”, in order to characterize the change of state or control input and the stability derivatives. The stability derivatives are expressed as the partial derivatives for the aerodynamic force, Equation (19), and moment, Equation (20), both sets of equations are expressed in terms of the vehicle coordinate system.

\[
\begin{align*}
\Delta F_x &= \frac{\partial F_x}{\partial u} \Delta u + \frac{\partial F_x}{\partial w} \Delta w + \frac{\partial F_x}{\partial \delta_c} \Delta \delta_c + \frac{\partial F_x}{\partial \delta_T} \Delta \delta_T \\
\Delta F_y &= \frac{\partial F_y}{\partial w} \Delta w + \frac{\partial F_y}{\partial \delta_T} \Delta \delta_T \\
\Delta F_z &= \frac{\partial F_z}{\partial u} \Delta u + \frac{\partial F_z}{\partial w} \Delta w + \frac{\partial F_z}{\partial q} \Delta q + \frac{\partial F_z}{\partial \delta_e} \Delta \delta_e + \frac{\partial F_z}{\partial \delta_T} \Delta \delta_T
\end{align*}
\]

(19)

Where : \( \sum F = \sum (F_0 + \Delta F) \)

\[
\begin{align*}
\Delta L &= \frac{\partial L}{\partial u} \Delta u + \frac{\partial L}{\partial w} \Delta w + \frac{\partial L}{\partial \delta_\alpha} \Delta \delta_\alpha + \frac{\partial L}{\partial \delta_T} \Delta \delta_T \\
\Delta M &= \frac{\partial M}{\partial u} \Delta u + \frac{\partial M}{\partial w} \Delta w + \frac{\partial M}{\partial \delta_e} \Delta \delta_e + \frac{\partial M}{\partial \delta_T} \Delta \delta_T \\
\Delta N &= \frac{\partial N}{\partial u} \Delta u + \frac{\partial N}{\partial w} \Delta w + \frac{\partial N}{\partial \delta_r} \Delta \delta_r + \frac{\partial N}{\partial \delta_T} \Delta \delta_T
\end{align*}
\]

(20)

Where : \( \sum M = \sum (M_0 + \Delta M) \)

The stability derivatives for the vehicle coordinate system can be transformed to the stability coordinate system via the following relationships\(^{11} \) in Equation (21), where \( \mathcal{F} \) represents any function.

\[
\begin{align*}
\frac{\partial \mathcal{F}}{\partial u} &= \frac{\partial \mathcal{F}}{\partial V_T} \\
\frac{\partial \mathcal{F}}{\partial v} &= \frac{\partial \mathcal{F}}{\partial \beta} \\
\frac{\partial \mathcal{F}}{\partial w} &= \frac{\partial \mathcal{F}}{\partial \alpha}
\end{align*}
\]

(21)

2. Perturbation Analysis

For characterization of vehicle behavior with respect to changes in states or control deflections from a designated trim point, a simple perturbation analysis was conducted as documented by Duke et al.\(^{14} \). With the assumption of small deviations, neighboring state solutions can be found from the original state solution and the input or state deviation using a a first-order linear differential equation. The nine nonlinear equations in Equation (18) are expanded using a Taylor-Series expansion about the initial states, \( x_0, \dot{x}_0, \) and initial controls, \( u_0 \). The state and control vectors are replaced with the initial states and control vectors with a small deviation added to each state and control shown by Equation (22).

\[
\begin{align*}
\dot{x} (t) &= x_0 (t) + \Delta x (t) \\
\dot{u} (t) &= u_0 (t) + \Delta u (t)
\end{align*}
\]

(22)

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Equation (22) is substituted into the differential equations of Equation (18) then expanded via a Taylor Series about the trim states resulting in Equation (23).

$$\dot{x}_i(t) = f_i(x_0(t), \dot{x}(t), u_0(t)) + \left. \frac{\partial f_i}{\partial x} \right|_{x=x_0, u=u_0} \Delta x + \left. \frac{\partial f_i}{\partial x} \right|_{x=x_0, u=u_0} \dot{\Delta x} + \left. \frac{\partial f_i}{\partial u} \right|_{x=x_0, u=u_0} \Delta u + \text{Higher Order Terms}$$

Evaluation relative to trim flight conditions allows for the accelerations of the states, \(\ddot{x}\), resulting from the force and moment equations to be zero so that the state velocities, \(\dot{x}\), are constant. The higher-order terms are neglected due to the assumption of the general solution not deviating wildly with respect to the particular solution. The resulting linearized state deviation equation of a second-order differential equation accurate to first-order is shown by Equation (24).

$$\Delta \dot{x} \approx \left( \frac{\partial f_i}{\partial x} \right)^{-1} \left( \frac{\partial f_i}{\partial x} \Delta x + \frac{\partial f_i}{\partial u} \Delta u \right)$$

Equation (24) provides a correction to the rates of the states which can then be integrated to form correction to the actual state values. This allows for nonlinear equations of motion to be linearized about a known trim point, permitting application of system identification analysis using state-space theory whereby a single \(n^{th}\)-order differential equation is reduced to \(n\) first-order differential equations.

**C. State-Space Representation of Vehicle Motion**

The partial derivatives in Equation (24) are known as Jacobian matrices and are defined in the following way:

$$\frac{\partial F_i}{\partial y} = \begin{bmatrix} \frac{\partial F_1}{\partial y_1} & \cdots & \frac{\partial F_1}{\partial y_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_m}{\partial y_1} & \cdots & \frac{\partial F_m}{\partial y_n} \end{bmatrix} \bigg|_{x=x_0, u=u_0}$$

Where:

$$\frac{\partial F_m}{\partial y_p} = \frac{F_m(y_{p,0} + \Delta y) - F_m(y_{p,0} - \Delta y)}{2\Delta y_p}$$

with \(m, p \in [1, 9]\)

Expressing the Jacobian with respect to the rate of change of the states \([E]\), the Jacobian with respect to the states \([A]\), and the Jacobian with respect to the control inputs \([B]\), Equation (24) becomes Equation (26).

$$\Delta \dot{x} = [E]^{-1} [A] \Delta x + [E]^{-1} [B] \Delta u$$

$$= [A]_{LIN} \Delta x + [B]_{LIN} \Delta u$$

Using shorthand notation in the perturbation analysis, the \(\Delta \dot{x}\), \(\Delta x\), and \(\Delta u\) are replaced with \(\dot{x}, x,\) and \(u\) respectively.

**1. Longitudinal Motion**

From the perturbation analysis assumptions the following linearized state-space model in stability coordinates is generated for longitudinal motion comprised of four states and controlled by two control effectors. The longitudinal states are: total velocity, \(V_T\), angle-of-attack, \(\alpha\), pitch rate, \(p\), and vehicle pitch angle, \(\theta\). Note that vehicle pitch angle, \(\theta\), may not be directly equivalent to the Euler elevation angle, \(\Theta\), due to vehicle roll and yaw displacements. The state-space model presented in Equation (27) as derived is accurate to first-order, the stability derivatives are given by Phillips11 with \(m\) being mass, \(I_{yyb}\) equal to vehicle y-axis inertia, and \(V_0\) being initial trim velocity. Small angle assumptions are made such that if the angle, \(\gamma\) is very small then \(\cos \gamma \approx 1\) and \(\sin \gamma \approx \gamma\).
\[ \Delta x_{\text{long}} = [\Delta V_T, \Delta \alpha, \Delta q, \Delta \theta]^T \]
\[ \Delta u_{\text{long}} = [\Delta \delta_T, \Delta \delta_e]^T \]

\[
[E]_{\text{long}} = \begin{bmatrix}
m & 0 & 0 & 0 \\
0 & m - F_{zb,\dot{w}}V_0 & 0 & 0 \\
0 & -M_{yb,\dot{w}}V_0 & I_{yb} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
[A]_{\text{long}} = \begin{bmatrix}
F_{zb,V_T} & F_{zb,a}V_0 & 0 & -mg \\
F_{zb,V_T} & F_{zb,a}V_0 & F_{zb,q} + V_0m & -mg (\theta_0 - \alpha_0) \\
M_{yb,V_T} & M_{yb,a}V_0 & M_{yb,q} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
[B]_{\text{long}} = \begin{bmatrix}
F_{xb,V_T} & F_{xb,a} \\
F_{xb,V_T} & F_{xb,r} \\
M_{yb,V_T} & M_{yb,a} \\
0 & 0
\end{bmatrix}
\]

\[ \Delta x_{\text{latd}} = [\Delta \beta, \Delta p, \Delta r, \Delta \phi, \Delta \psi]^T \]
\[ \Delta u_{\text{latd}} = [\Delta \delta_u, \Delta \delta_v]^T \]

\[
[E]_{\text{latd}} = \begin{bmatrix}
m & 0 & 0 & 0 & 0 \\
0 & I_{xz} & -I_{xz} & 0 & 0 \\
0 & -I_{xz} & I_{xz} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad [B]_{\text{latd}} = \begin{bmatrix}
F_{yb,\delta_u} & F_{yb,\delta_v} \\
M_{zb,\delta_u} & M_{zb,\delta_v} \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

\[ \Delta x_{\text{latd}} = [\Delta \beta, \Delta p, \Delta r, \Delta \phi, \Delta \psi]^T \]
\[ \Delta u_{\text{latd}} = [\Delta \delta_u, \Delta \delta_v]^T \]

\[
[A]_{\text{latd}} = \begin{bmatrix}
F_{zb,\delta_v}/V_0 & F_{zb,a}V_0 & F_{zb,a} - mV_0 & mg & 0 \\
M_{zb,\delta_v}/V_0 & M_{zb,p} & M_{zb,q} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

III. Observer/Kalman Filter Identification

Based on the concept of stochastic Kalman filter estimation and the techniques of deterministic Markov parameter identification, OKID generates a time-domain discrete state-space model representation. For lightly damped systems and modes, such as the phugoid mode of longitudinal aircraft dynamics, OKID can artificially improve system damping, making the system deadbeat after just a few steps. This significantly reduces the required data record, storage space, and computation time. In practice, ideal linear system identification is almost impossible when external disturbances, often unknown, act on the system. A variant of the OKID algorithm developed to solve this problem by Chen and Valasek is applied to the Extra 300.
flight test data. The discrete state-space perturbation model of the trimmed nonlinear vehicle dynamics is assumed to have the form:

$$X(k + 1) = AX(k) + Bu(k)$$
$$y(k) = CX(k) + Du(k)$$ (29)

where \(X(k) \in \mathbb{R}^n, y(k) \in \mathbb{R}^m, u(k) \in \mathbb{R}^r\), are state, output and control inputs with dimension of \(n, m, \) and \(r\) respectively. Following the development of Juang,\(^8\) solving for the output \(y(k)\) with zero initial condition from Equation (29) in terms of the previous inputs \(u(i)(i = 0, 1, 2, \cdots, k)\) yields

\[
\begin{align*}
X(0) &= 0, \\
y(0) &= Du(0) ; \\
X(1) &= Bu(0), \\
y(1) &= CBu(0) + Du(1); \\
X(2) &= ABu(0) + Bu(1), \\
y(2) &= CABu(0) + CBu(1) + Du(2); \\
& \vdots \\
X(k) &= \sum_{i=1}^{k} A^{i-1} Bu(k-i) \\
y(k) &= \sum_{i=1}^{k} CA^{i-1} Bu(k-i) + Du(k)
\end{align*}
\] (30)

Writing Equation (30) in matrix form, we have

\[
y = [y(0) \ y(1) \ y(2) \cdots y(l-1)] \\
Y = [D \ CB \ CAB \cdots CA^{l-2}B ]
\] (31)

where

\[
U = \begin{bmatrix}
u(0) & u(1) & u(2) & \cdots & u(l-1) \\
u(0) & u(1) & \cdots & u(l-2) \\
u(0) & \cdots & u(l-3) \\
\vdots \\
u(0)
\end{bmatrix}
\] (32)

and \(D, CB, CAB, \cdots, CA^{k-1}B\) are called Markov Parameters, and are commonly used as the basis to identify mathematical models for linear dynamic systems. Markov parameters generate state-space models by forming a Hankel matrix.\(^8\) For non-zero initial conditions, the following derivation is used:

\[
\begin{align*}
x(k + 1) &= \bar{A}x(k) + \bar{B}v(k), \\
x(k + 2) &= \bar{A}x(k + 1) + \bar{B}v(k + 1) \\
&= \bar{A}^2x(k) + \bar{A}Bv(k) + \bar{B}v(k + 1), \\
& \vdots \\
x(k + p) &= \bar{A}x(k + p - 1) + \bar{B}v(k + p - 1)
\end{align*}
\] (33)

Note that in Equation (33), \(v(k)\) is the input vector to the new augmented system. These equations can be written as

\[
x(k + p) = \bar{A}^p x(k) + \bar{A}^{p-1}\bar{B}v(k) + \bar{A}^{p-2}\bar{B}v(k + 1) \\
+ \cdots + \bar{B}v(k + p - 1)
\] (34)
using the measurement equation yields

\[ y(k+p) = Cx(k+p) + Du(k+p) \]
\[ = CA^p x(k) + CA^{p-1} Bv(k) + CA^{p-2} Bv(k+1) + \cdots + CA Bv(k+p-1) + Du(k+p) \]  

(35)

The set of these equations for a sequence of \( k = 0, \cdots, l - 1 \) can be written as

\[ \bar{y} = CA^p x + Y \bar{v} \]  

(36)

where

\[ \bar{y} = [y(p) \ y(p+1) \ \cdots y(l-1)] \]
\[ \bar{Y} = [D \ CB \ C \bar{A}B \ \cdots \ C\bar{A}^{(p-1)}\bar{B}] \]
\[ \bar{V} = \begin{bmatrix} u(p) & u(p+1) & \cdots & u(l-1) \\ v(p-1) & v(p) & \cdots & v(l-2) \\ v(p-2) & v(p-1) & \cdots & v(l-3) \\ \vdots & \vdots & \cdots & \vdots \\ v(0) & v(1) & \cdots & v(l-p-1) \end{bmatrix} \]  

(37)

and \( D, C\bar{B}, C\bar{A}B, \cdots, C\bar{A}^{(p-1)}\bar{B} \) are the Observer Markov parameters. Through manipulation, system Markov parameters can then be recovered from the observer Markov parameters \( \bar{Y} \) through partition of \( \bar{Y} \) as:

\[ \bar{Y} = [D \ CB \ C\bar{A}B \ \cdots \ C\bar{A}^{(p-1)}\bar{B}] = [\bar{Y}_0 \ \bar{Y}_1 \ \bar{Y}_2 \ \cdots \ \bar{Y}_p] \]  

(38)

from which observer Markov parameters are obtained.

\[ \bar{Y}_0 = D \]
\[ \bar{Y}_k = C\bar{A}^{(k-1)}\bar{B} \]
\[ = [C(A + GC)^{k-1} (B + GD) - C(A + GC)^{k-1}G] \]
\[ = [\bar{Y}_k^{(1)} - \bar{Y}_k^{(2)}]; \ k = 1, 2, 3, \ldots \]  

(39)

Based on the definition of system Markov parameters, the system Markov parameters are

\[ Y_1 = CB = C(B + GD) - (CG)D \]
\[ = \bar{Y}_1^{(1)} - \bar{Y}_1^{(2)} D \]
\[ Y_2 = C\bar{A}B \]
\[ = \bar{Y}_2^{(1)} - \bar{Y}_1^{(2)} Y_1 - \bar{Y}_2^{(2)} D \]
\[ \vdots \]

(40)

According to the derivation above, the general relationship between the actual system Markov parameters and the observer Markov parameters is

\[ D = Y_0 = \bar{Y}_0 \]
\[ Y_k = Y_k^{(1)} - \sum_{i=1}^{k} \bar{Y}_i^{(2)} Y_{(k-i)} \text{ for } k = 1, \cdots, p \]
\[ Y_k = -\sum_{i=1}^{p} \bar{Y}_i^{(2)} Y_{(k-i)} \text{ for } k = p + 1, \cdots, \infty \]  

(41)

Now the desired discrete system realization \( [A, B, C, D] \) is obtained from the system Markov parameters from a singular value decomposition (SVD) of the Hankel matrix.

\[ H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \cdots & Y_{k+\alpha+\beta-2} \end{bmatrix} \]
\[ H(0) = R_\alpha \Sigma S_n \]  

(42)
\[ \hat{A} = \Sigma^{-1/2} R^T_n H(1) \Sigma^{-1/2} \]
\[ \hat{B} = \Sigma^{1/2} S^T_n E_r \]
\[ \hat{C} = E^T_m R_n \Sigma^{-1/2} \]  
(43)

where

\[ E^T_m = [I_m \ O_m \ \cdots \ O_m], \quad E^T_r = [I_r \ O_r \ \cdots \ O_r] \]  
(44)

With the control inputs and perturbation outputs data, the perturbation linear model can be identified using a OKID approach:

\[ X(k+1) = AX(k) + Bu(k) \]
\[ y(k) = X(k) \]  
(45)

IV. Application of OKID to Extra 300 UAV

The OKID algorithm\(^8\) was applied to the Extra 300 UAV test-bed flight data for identification of vehicle behavior resulting from control effectors. The main focus of this research is quantitatively compare different roll control effectors for lateral/directional motion. The identified longitudinal models and stability analysis is presented for completeness. For the longitudinal and lateral/directional character assessment the Extra 300 flew with a constant weight, fuselage shape, along with wing area and shape characteristics for appropriate trial comparison. The conventional and newly proposed jet-actutators cannot be directly compared due discrepancies in the latter’s vehicle inertia which alters its roll mode characteristics drastically. The control input magnitudes and duration were arrived at through trial and error so as to excite the relevant system modes without forcing the aircraft outside of the linear operating regime.

A. Longitudinal Motion

For longitudinal motion identification, the Extra 300 was brought to a trim flight state then excited via elevator doublet maneuvers to characterize behavior. Separate trials were performed to assess vehicle longitudinal character due to positive then negative elevator input, Trial 1, and negative then positive elevator input, Trial 2. A positive elevator input corresponds to a negative, restoring, moment about the vehicle secondary axis.

For Trial 1, a singular value decomposition analysis was performed to estimate the system order and shows the model for the longitudinal motion should be fourth order as illustrated by Figure 1, a similar result was found for Trial 2. Figure 1 shows the singular values tend to flatten out after four values with the largest singular value being approximately 1 and the fourth largest singular value being approximately $10^{-7}$. Thus, the data reinforces the assumption of the longitudinal motion being represented by a state-space model comprised of four states, a single fourth order differential equation will describe the linearized model.

![Figure 1. Singular Value Decomposition for Longitudinal Trial 1 Analysis](image-url)
Proposing a fourth order model with the measured control inputs shown by Figure 2 and measured outputs, the OKID algorithm produced the following continuous system properties listed in Table 1.

The eigenvalues for Trials 1 and 2 in the table illustrate the presence of different modes characterized by their natural frequency/damping distribution. Trial 1 displays results close to what one would expect, where normally longitudinal motion is governed by two oscillatory modes: (1) a lightly damped, long period mode governed by the exchange of translational energy named the phugoid mode, and (2) a heavily damped, short period mode governed by the exchange of rotational energy.\(^{11}\)

<table>
<thead>
<tr>
<th>Trial</th>
<th>Eigenvalue 1</th>
<th>Damping, (\zeta)</th>
<th>Natural Frequency, (\omega_n), (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-13.2340</td>
<td>1</td>
<td>13.2340</td>
</tr>
<tr>
<td></td>
<td>-7.5647</td>
<td>1</td>
<td>7.5647</td>
</tr>
<tr>
<td></td>
<td>-2.0273±1.1564i</td>
<td>0.8686</td>
<td>2.3339</td>
</tr>
<tr>
<td>2</td>
<td>-21.1742</td>
<td>1</td>
<td>21.1742</td>
</tr>
<tr>
<td></td>
<td>-3.2075</td>
<td>1</td>
<td>3.2075</td>
</tr>
<tr>
<td></td>
<td>-0.8165</td>
<td>1</td>
<td>0.8165</td>
</tr>
<tr>
<td></td>
<td>-0.5117</td>
<td>1</td>
<td>0.5117</td>
</tr>
</tbody>
</table>

Table 1. Continuous Properties for Eigenvalues of Identified Linearized Longitudinal \([A]_{\text{Lin,Long}}\) Matrix for Trials 1 & 2

Tables 2 and 3 compare the identified discrete \([A]_{\text{Lin,Long}}\) and \([B]_{\text{Lin,Long}}\) matrices from the OKID algorithm for Trials 1 and 2. Since the identified system models from OKID are discrete the stability derivatives cannot be directly calculated from the matrix entries, however a comparison between the identified magnitudes between trials can be performed. The continuous eigenvalues in Table 1 were calculated by taking the natural logarithm of the discrete eigenvalues divided by the sampling time.

Figure 3 displays the error between the recorded flight output states of vehicle pitch angle and pitch rate and the simulation results of the Trial 1 state-space model for the given system inputs shown by Figure 2. Figure 4 displays the error between the recorded flight output states of angle-of-attack and total velocity and simulation results of the Trial 1 state-space model for the same system inputs. The error plots for Trial 2 closely resemble Trial 1 so they are omitted. The plots show a sub-one degree error and sub-meter per second error for the system states, however the error magnitude value is extremely sensitive to the OKID fit interval and input magnitudes.
<table>
<thead>
<tr>
<th>Trial</th>
<th>( V_T ) State</th>
<th>( \alpha ) State</th>
<th>( q ) State</th>
<th>( \theta ) State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6951</td>
<td>-0.0210</td>
<td>-0.0001</td>
<td>0.00005</td>
</tr>
<tr>
<td></td>
<td>0.46971</td>
<td>0.9162</td>
<td>-0.0010</td>
<td>0.00008</td>
</tr>
<tr>
<td></td>
<td>-0.0291</td>
<td>0.0085</td>
<td>0.9568</td>
<td>0.00445</td>
</tr>
<tr>
<td></td>
<td>-0.1602</td>
<td>0.4000</td>
<td>-0.4641</td>
<td>1.03370</td>
</tr>
<tr>
<td>2</td>
<td>0.6279</td>
<td>-0.0032</td>
<td>0.0001</td>
<td>0.00024</td>
</tr>
<tr>
<td></td>
<td>0.4659</td>
<td>0.9380</td>
<td>0.0006</td>
<td>-0.00164</td>
</tr>
<tr>
<td></td>
<td>-0.3946</td>
<td>0.8314</td>
<td>0.9916</td>
<td>0.05372</td>
</tr>
<tr>
<td></td>
<td>-0.1230</td>
<td>0.2604</td>
<td>-0.0082</td>
<td>0.98384</td>
</tr>
</tbody>
</table>

Sample Time \( \Delta t = 0.020983 \)

Table 2. Identified Linearized Discrete \([A]_{Lin,Long}\) Matrix for Trials 1 & 2

<table>
<thead>
<tr>
<th>Trial</th>
<th>( \delta_T ) Input</th>
<th>( \delta_e ) Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.4523</td>
<td>2.4935</td>
</tr>
<tr>
<td></td>
<td>0.2881</td>
<td>-1.3484</td>
</tr>
<tr>
<td></td>
<td>0.4524</td>
<td>0.9304</td>
</tr>
<tr>
<td></td>
<td>0.4049</td>
<td>1.5923</td>
</tr>
<tr>
<td>2</td>
<td>-0.5440</td>
<td>2.8599</td>
</tr>
<tr>
<td></td>
<td>0.2468</td>
<td>-1.3097</td>
</tr>
<tr>
<td></td>
<td>-0.0605</td>
<td>1.7297</td>
</tr>
<tr>
<td></td>
<td>-0.4790</td>
<td>-0.4770</td>
</tr>
</tbody>
</table>

Sample Time \( \Delta t = 0.020983\) seconds

Table 3. Identified Linearized Discrete \([B]_{Lin,Long}\) Matrix for Trials 1 & 2

Figure 3. Error Comparison of Vehicle Pitch Displacement and Rate versus Trial 1 Linearized State-Space Model

B. Lateral/Directional Motion

Following the same test evaluation as the longitudinal trials, the lateral/ directional modes were examined using a combination of aileron and rudder doublets for system identification using the OKID algorithm. The lateral/ directional motion evaluations consisted of three configurations: (1) Full Ailerons, inboard and outboard, (2) Reduced Aileron, inboard only, and (3) Jet-Actuators replacing the outboard conventional
ailerons. The weight of configurations (1) and (2) were modified by adding ballast to replicate the weight distribution of jet-actuated configuration. The wing surface area, shape, and airfoil profile were consistent between all three configurations. However, the plant inertia differences between the conventional and jet-actuated platform were significant enough to alter plant dynamics so direct one-to-one comparison between conventional ailerons and jet-actuators cannot be done. Additionally, data for the jet-actuated configuration with inboard ailerons was not recorded so a comparison between inboard aileron control and jet-actuated cannot be performed.

1. Full Aileron Configuration

A total of five trials were run using the conventional Extra 300 configuration with full use of the conventional inboard and outboard ailerons with four producing good, stable simulation recreation, OKID results via control input magnitudes not forcing the aircraft into a nonlinear response regime. The total time interval to complete the rudder and aileron doublet pair was approximately four seconds each time with the aileron doublet coming first, Figure 5 shows representative inputs to the aircraft.

Following the same analysis as the longitudinal, the measured input and output response data was subjected to singular value decomposition analysis showing the system is dominated by four singular values. This is appropriate because the vehicle yaw displacement state, $\psi$, is a redundant state and is only the integral of the yaw rate whereas the vehicle roll angle, $\phi$, is dependent on roll and yaw rate. The singular value plot is shown in Figure 6 for a trial chosen to be representative of the five test runs showing the order of the model.
should be four due to the large gap between the fourth and fifth largest singular values. The trial chosen to be representative of the full aileron configuration has its continuous system characteristics shown in Table 4.

![Hankel Matrix Singular Values]

**Figure 6. Singular Value Decomposition for Lateral/Directional Full Aileron Configuration Representative Trial**

<table>
<thead>
<tr>
<th>Number</th>
<th>SV Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10^{-6}</td>
</tr>
<tr>
<td>2</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>3</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>10^{-3}</td>
</tr>
<tr>
<td>5</td>
<td>10^{-2}</td>
</tr>
<tr>
<td>6</td>
<td>10^{-1}</td>
</tr>
<tr>
<td>7</td>
<td>10^{0}</td>
</tr>
</tbody>
</table>

**Table 4. Continuous Properties for Eigenvalues of Identified Linerized Lateral/Directional \([A]_{Lin, Long}\) Matrix for Full Aileron Configuration Representative Trial**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Damping, (\zeta)</th>
<th>Natural Frequency, (\omega_n), (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.0083</td>
<td>1</td>
<td>3.0083</td>
</tr>
<tr>
<td>-1.7744±6.0395(i)</td>
<td>0.2819</td>
<td>6.2948</td>
</tr>
<tr>
<td>-1.4070</td>
<td>1</td>
<td>1.4070</td>
</tr>
</tbody>
</table>

Table 4 is in agreement with conventional aircraft lateral/directional motion, typically characterized by two distinct roots and one complex pair. Additionally, the roll mode is characterized by the largest real eigenvalue which gives the roll period settling time, two percent, of approximately 1.2 seconds with the spiral mode characterized by the smallest real eigenvalue making the spiral settling time approximately 2.9 seconds. The roll mode is relatively quick because it is most dependent on the ailerons while the spiral mode is typically characterized by a period of a few seconds. The Dutch-Roll mode is characterized by the complex pair and represents the out of phase combination sideslip, roll, and yaw oscillations with an identified settling time of approximately 0.7 seconds. The quick response of the identified model is a good representation of the Extra 300 since the airframe is known to be extremely responsive. The response time of the aircraft is assumed to be a bit longer than a typical Extra 300 due to the increased weight of added ballast, batteries, and electronics. Table 5 presents the identified discrete state-space model using OKID with the measured input and output response histories. Only the \([A]_{Lin, Latd}\) and \([B]_{Lin, Latd}\) matrices are needed for simulation since the state observation matrix can be set to Identity without direct transmission.

The error plots of the representative trial for the full aileron configuration show the estimates of vehicle roll and yaw angular displacement fluctuate largely during the OKID fit interval. This is due to the yaw and roll rate measurements being in error and “chattering” significantly. As demonstrated by previous research, minute errors in angular rate measurement lead to significant error when integrated for angular displacement and must have their initial conditions reconciled repeatedly to produce accurate measurements for vehicle angular displacements. The electronics package installed on the Extra 300 test-bed does not incorporate additional inertial sensors to correct rate gyro errors during flight operation so the produced angular displacement measurements are not very accurate. However, the state-space simulation produces accurate results for the sideslip angle.
<table>
<thead>
<tr>
<th>Matrix</th>
<th>State</th>
<th>p State</th>
<th>r State</th>
<th>φ State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} A \end{bmatrix}_{Lin, Latd}$</td>
<td>0.9506</td>
<td>0.0391</td>
<td>-0.0115</td>
<td>0.0129</td>
</tr>
<tr>
<td></td>
<td>-0.0733</td>
<td>0.9318</td>
<td>-0.0247</td>
<td>0.0269</td>
</tr>
<tr>
<td></td>
<td>0.3087</td>
<td>0.1083</td>
<td>0.9506</td>
<td>-0.0120</td>
</tr>
<tr>
<td></td>
<td>0.1155</td>
<td>-0.3103</td>
<td>-0.0473</td>
<td>0.9882</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix</th>
<th>δ_δ Input</th>
<th>δ_r Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} B \end{bmatrix}_{Lin, Latd}$</td>
<td>1.207</td>
<td>0.4028</td>
</tr>
<tr>
<td></td>
<td>-0.4148</td>
<td>1.0730</td>
</tr>
<tr>
<td></td>
<td>-1.3520</td>
<td>-0.8534</td>
</tr>
<tr>
<td></td>
<td>-1.1770</td>
<td>0.9277</td>
</tr>
</tbody>
</table>

Sample Time $\Delta t = 0.020983$ seconds

Table 5. Identified Linearized Discrete $[A]_{Lin, Latd}$ and $[B]_{Lin, Latd}$ Matrices for Full Aileron Configuration Representative Trial

![Graph](https://example.com/graph1.png)

Figure 7. Error Comparison of Vehicle Sideslip Angle versus Lateral/Directional Full Aileron Configuration Representative Trial State-Space Model

![Graph](https://example.com/graph2.png)

Figure 8. Error Comparison of Vehicle Roll Angle and Roll Rate versus Lateral/Directional Full Aileron Configuration Representative Trial State-Space Model

2. Reduced Aileron Configuration

A reduced aileron configuration, only the inboard wing ailerons, was conducted to provide a lower bounding for the aileron control effectiveness since the full aileron configuration yielded an upper bound for the
conventional wing control authority. Two trials were conducted for evaluation of lateral/directional motion of the aircraft using the inboard ailerons while the outboard ailerons were kept at their trim settings. The first trial consisted of a rudder then aileron doublet while the second was an aileron then rudder doublet, the input displacements from trim are shown in Figure 10. The continuous time properties identified using the OKID algorithm, Table 6, are consistent with one another and followed the same eigenvalue distribution as the full aileron representative configuration. The singular value analysis for the data revealed the same trend as shown in the full aileron case where the appropriate system model is of fourth order. The identified discrete state-space model matrices compared for each trial are given by Tables 7 and 8.

Comparison of Table 8 with Table 5 for the discrete $[B]_{Lin,Lat}$ shows aileron control effectiveness is indeed reduced by not allowing use of the outboard ailerons. This is an intuitive result since the outboard ailerons contribute a larger rolling moment due to their larger distance from the center-of-gravity and fuselage. Comparison of Table 7 with Table 5 for the discrete $[A]_{Lin,Lat}$ matrix shows that the reduced aileron configuration and full aileron configuration are the same plant model since the identified plant matrices are very similar to each other. Comparison of the aileron authorities between the full and reduced aileron configuration show that the reduced aileron configuration possesses a higher authority over vehicle roll angle acceleration. This is attributed to the need for a larger deflection of the inboard ailerons to properly excite the system for identification where the required deflection was approximately three times that of the full aileron configuration, compare Figures 5 and 10.

The following figures display the error between the recorded flight output data and the simulation out-
<table>
<thead>
<tr>
<th>Trial</th>
<th>Eigenvalue</th>
<th>Damping, ζ</th>
<th>Natural Frequency, ω_n (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5.2090</td>
<td>1</td>
<td>5.2090</td>
</tr>
<tr>
<td></td>
<td>-2.2298±6.975i</td>
<td>0.3045</td>
<td>7.3228</td>
</tr>
<tr>
<td></td>
<td>-0.1762</td>
<td>1</td>
<td>0.1762</td>
</tr>
<tr>
<td>2</td>
<td>-5.8735</td>
<td>1</td>
<td>5.8735</td>
</tr>
<tr>
<td></td>
<td>-1.9112±6.674i</td>
<td>0.2753</td>
<td>6.9424</td>
</tr>
<tr>
<td></td>
<td>-0.4197</td>
<td>1</td>
<td>0.4197</td>
</tr>
</tbody>
</table>

Table 6. Continuous Properties for Eigenvalues of Identified Linearized Lateral/Directional [A]_{Lin,Long} Matrix for Reduced Aileron Configuration Trials

<table>
<thead>
<tr>
<th>Trial</th>
<th>β State</th>
<th>p State</th>
<th>r State</th>
<th>φ State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8886</td>
<td>-0.0140</td>
<td>-0.0704</td>
<td>0.0052</td>
</tr>
<tr>
<td></td>
<td>-0.0725</td>
<td>0.8821</td>
<td>-0.0789</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td>0.2767</td>
<td>0.0870</td>
<td>1.0260</td>
<td>-0.0073</td>
</tr>
<tr>
<td></td>
<td>0.137</td>
<td>-0.2999</td>
<td>-0.0634</td>
<td>0.9843</td>
</tr>
<tr>
<td>2</td>
<td>0.8877</td>
<td>0.0122</td>
<td>-0.0630</td>
<td>-0.0257</td>
</tr>
<tr>
<td></td>
<td>-0.0374</td>
<td>0.9546</td>
<td>0.0109</td>
<td>-0.0061</td>
</tr>
<tr>
<td></td>
<td>0.3164</td>
<td>0.0244</td>
<td>0.9966</td>
<td>0.0338</td>
</tr>
<tr>
<td></td>
<td>0.0104</td>
<td>-0.3414</td>
<td>0.0075</td>
<td>0.9389</td>
</tr>
</tbody>
</table>

Table 7. Identified Linearized Discrete [A]_{Lin,Latd} Matrices for Reduced Aileron Trials 1 & 2

Table 8. Identified Linearized Discrete [B]_{Lin,Latd} Matrices for Reduced Aileron Trials 1 & 2

Put from the identified system models using the recorded flight input data. The OKID fit period for Trial 1 was 4 seconds while the fit period for Trial 2 was 4.5 seconds. The errors in the angular displacement and angular rate states are plotted the same was as in the full aileron configuration with the trials overlaid for ease of comparison. In Figure 11, sideslip angle, has the smallest error followed by vehicle yaw angle, Figure 13, then vehicle roll angle, Figure 12, which follows the same error trend of the full aileron configuration tests.
3. Jet-Actuated Configuration

The jet-actuated control configuration was installed on the Extra 300 UAV test-bed by replacing the conventional wing setup with the same wing configuration, shape and size, using inboard ailerons but with...
jet-actuated outboard control effectors in lieu of conventional ailerons. The aircraft weight was the same as in the full and reduced aileron configurations since the ballast used in the wings for those configurations was used to simulate the jet-actuated wing configuration. The weight distribution proved to not be constant between the conventional wings and jet-actuated wings leading to different plant dynamics and the inability to compare control authority one-to-one. Three trials were performed with only two allowing for easily identifiable models due to OKID algorithm’s sensitivity to the fit interval and location of the interval with respect to the start and end of the control input. Trial 1 was approximately 4.5 seconds while Trial 2 was approximately 4 seconds. Both trials had a jet-actuated doublet and then a rudder doublet as shown in Figure 14. Singular value decomposition analysis confirmed the order model to be the same as the full and reduced aileron configurations. Table 9 lists the continuous system properties of the aircraft plant for lateral/directional motion.

![Figure 14. Jet-Actuator and Rudder Input for Lateral/Directional Jet-Acutated Configuration Trials 1 & 2](image)

<table>
<thead>
<tr>
<th>Trial</th>
<th>Eigenvalue</th>
<th>Damping, $\zeta$</th>
<th>Natural Frequency, $\omega_n$, (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-27.2280</td>
<td>1</td>
<td>27.2280</td>
</tr>
<tr>
<td></td>
<td>-2.3271±6.3171i</td>
<td>0.3456</td>
<td>6.7321</td>
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<tr>
<td>2</td>
<td>-22.3150</td>
<td>1</td>
<td>22.3150</td>
</tr>
<tr>
<td></td>
<td>-2.8713.</td>
<td>1</td>
<td>2.8713</td>
</tr>
</tbody>
</table>

Table 9. Continuous Properties for Eigenvalues of Identified Linearized Lateral/Directional $[A]_{Lin,Long}$ Matrix for Jet-Actuated Configuration Trials

From Table 9, the Dutch-Roll characteristics of the jet-actuated configuration aligns closely with those of the full and reduced aileron configurations. This is due to the Dutch-Roll mode being primarily dependent upon vertical tail size which is constant across the configuration because only the wings are switched. The roll and spiral mode roots of the jet-actuated configuration are much different that those of the conventional configurations and are due to the variations of the distribution of the weight in the two different wings even though they both have the same shape features and total weight. Of interest is the divergent root of Trial 2 corresponding to the spiral mode, however it is not uncommon for this root to be slightly divergent\textsuperscript{11} and is not considered too dangerous as long as the pilot is aware of its effects. The identified discrete state-space model matrices for each trial are given by Tables 10 and 11.

From comparison of the $[A]_{Lin,Latd}$ within the jet-actuated trials, the plant model appears consistent as does the control effectiveness for both the jet actuators and rudder. Comparison of the $[A]_{Lin,Latd}$ for the jet-actuated configuration trials versus the conventional full and reduced aileron plant matrices shows the
Table 10. Identified Linearized Discrete $[A]_{\text{Lin,Latd}}$ Matrices for Jet-Actuated Configuration Trials 1 & 2

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\beta$ State</th>
<th>p State</th>
<th>r State</th>
<th>$\phi$ State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5575</td>
<td>-0.0075</td>
<td>-0.0628</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>0.0572</td>
<td>0.8938</td>
<td>-0.0509</td>
<td>0.0025</td>
</tr>
<tr>
<td></td>
<td>0.1126</td>
<td>0.2696</td>
<td>1.0100</td>
<td>0.0079</td>
</tr>
<tr>
<td></td>
<td>0.3608</td>
<td>-0.1772</td>
<td>-0.0320</td>
<td>0.9392</td>
</tr>
<tr>
<td>2</td>
<td>0.6243</td>
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<td>0.0189</td>
</tr>
<tr>
<td></td>
<td>0.0262</td>
<td>0.8879</td>
<td>-0.0604</td>
<td>0.0051</td>
</tr>
<tr>
<td></td>
<td>0.1283</td>
<td>0.2665</td>
<td>1.012</td>
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</tr>
<tr>
<td></td>
<td>0.2961</td>
<td>-0.1708</td>
<td>0.0011</td>
<td>0.9963</td>
</tr>
</tbody>
</table>

Sample Time $\Delta t = 0.020983$ seconds

Table 11. Identified Linearized Discrete $[B]_{\text{Lin,Latd}}$ Matrices for Jet-Actuated Configuration Trials 1 & 2

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\delta_{JA}$ Input</th>
<th>$\delta_{r}$ Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.8850</td>
<td>-0.9269</td>
</tr>
<tr>
<td></td>
<td>0.5568</td>
<td>-1.0540</td>
</tr>
<tr>
<td></td>
<td>-0.3097</td>
<td>0.9246</td>
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<tr>
<td></td>
<td>0.7401</td>
<td>-0.2255</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td></td>
<td>0.3053</td>
<td>-1.194</td>
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<tr>
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<td>-0.0763</td>
<td>0.9257</td>
</tr>
<tr>
<td></td>
<td>0.7207</td>
<td>-0.4116</td>
</tr>
</tbody>
</table>

Sample Time $\Delta t = 0.020983$ seconds

Comparison of the $[B]_{\text{Lin,Latd}}$ of the jet-actuated models versus the reduced aileron configuration shown in Table 8 demonstrates that the jet-actuator plant and actuators have much larger control authority over the sideslip angle and yaw rate states and similar but larger control authority over the roll rate and roll angle states. Comparison of the jet-actuated control effectiveness versus the full aileron configuration in Table 5 shows the jet-actuated configuration was more effective in changing the sideslip angle and roll rate states but less effective in changing the yaw rate and roll angle states. These results demonstrate that the preconditioning of the jet-actuated plant significantly masks the effects of the control effectors due to the much larger plant roll frequency, approximately five times faster. Figures 15, 16, and 17 display the error between the recorded states and the simulated model states using the recorded input data. Again the sideslip state error, Figure 15, is the smallest among all angle states for the simulated models, followed by roll angle, Figure 16, then yaw angle, Figure 17. The large discrepancy in the roll and yaw angle is due to “chatter” of the rate gyro signals with integration of the minute rate gyro error resulting in displacement angle drift as the test interval proceeds. Figures 18, 19, 20 and 21 are included to provide state time history comparison for the angular rates for each trial showing the simulation appropriately tracks the recorded angular rates. These plots support the conclusion of the state angle error stemming from ill-
conditioned integration initial conditions and integration of minute errors which compound the error as the time interval progresses.
Figure 18. Comparison of Vehicle Recorded Roll Rate versus Lateral/Directional Jet-Actuated Configuration
Trial 1 State-Space Model Simulation

Figure 19. Comparison of Vehicle Recorded Roll Rate versus Lateral/Directional Jet-Actuated Configuration
Trial 2 State-Space Model Simulation

Figure 20. Comparison of Vehicle Recorded Yaw Rate versus Lateral/Directional Jet-Actuated Configuration
Trial 1 State-Space Model Simulation
V. Conclusion

This study provided a quantitative analysis of control effectiveness for a conventional aileron and jet-actuated configuration in lateral/directional motion. The observer/Kalman filter identification (OKID) algorithm was applied to identify locally linear state-space models for evaluation of controller effectiveness in lateral/directional motion for an Extra 300 remote controlled aircraft test-bed with longitudinal analysis provided for completeness. The test aircraft was used to assess lateral/directional control authority of conventional outboard ailerons and proposed flow-jet actuators. Differences in the plant wing weight distributions severely affected roll inertia and did not allow for a one-to-one actuator comparison since the plant dynamics of the jet-actuated configuration masked the true effectiveness of the control actuator. Roll actuator and rudder doublets were used to excite the system to form the input-output relationship needed to derive linearized lateral/directional equations of motion. The OKID algorithm in conjunction with singular value decomposition appropriately identified correct system model order and locally linear state-space models to accurately recreate system behavior given recorded control inputs. The only significantly large state errors in the study were for the vehicle angular displacements, roll, pitch, and yaw, and were attributed to the integration of the raw rate gyro signals without proper filtering since minute rate error integration can lead to large angular drifts if left uncorrected.

Linear system analysis was conducted using the identified discrete time state-space models from the OKID algorithm for the three different test-bed configurations: (1) Full Ailerons, (2) Reduced Ailerons, and (3) Jet-Actuators. The reduced aileron configuration set the lower bound for actuator characterization since the control surfaces on the outboard section of the wing were not utilized while the full aileron configuration provided the upper bound for actuator characterization. Comparison of the conventional aileron control configurations showed the reduced aileron configuration to possess a larger roll angular acceleration authority. This is attributed to the need for a larger deflection of the inboard ailerons, approximately three times greater than the outboard, to properly perturb the system for system identification.

The jet-actuated configuration was shown as an alternative for lateral/directional control but a direct comparison with conventional ailerons could not be performed due to dissimilar plant dynamics between the two control effector wing configurations. The jet-actuated plant possessed a much higher higher roll inertia which masked the control effectiveness while Dutch-Roll mode was unaffected since it is mainly a function of vertical tail size which along with the fuselage remained constant. The linear analysis showed the jet-actuated identified plant model provided better roll and spiral motion responses while having a similar Dutch-Roll motion response. The plant model difference is attributed primarily to the distribution of weight in the wing configurations since all configurations utilized the same wing shape characteristics and overall aircraft weight.
References


