Fault Tolerant Control Allocation for Mars Vehicle using Adaptive Control

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Problem Statement

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Adaptive guidance and control systems are a potential solution
Research Overview

**Approach:** To design a fault tolerant controller for Mars entry vehicle

**Key Issues:**
- Nonlinear time varying system
- Guided or Ballistic reference trajectory
- Uncertainties in aerodynamic coefficients, Mars atmospheric density, winds and inertia properties of the vehicle
Research Overview

**Solution:** To use fault tolerant discrete control allocation coupled with adaptive control

**Benefits:**
- Track both kinematics and dynamic level states
- Satisfactory tracking performance even in case of jet failures
- Handles uncertainties in aerodynamic coefficients, Mars atmospheric density and vehicle's inertia properties
Mars Ellipsled Video
Vehicle Model

Vehicle Model

• A scaled down model of Mars Ellipsled
• Cylinder with diameter 3.75 m and length 6.323 m

RCS Model

• Eighteen RCS control jets
• Nine jets on each side in clusters of three
• Each jet influence more than one axis
• Jet on/off time is not considered
• Provide constant thrust throughout flight
Trajectory

• **Vehicle Trajectory**

\[ J(\sigma) = 0.25[1 - \sigma^T \sigma]I_{3\times3} + 2[\tilde{\sigma}] + 2\sigma\sigma^T \]

\[ I \dot{\omega} + \omega \times I \omega = M_{\text{aerodynamic}} + M_{\text{control}} \]

where

\( \sigma \) Modified Rodrigues Parameters (MRPs)

\( \omega \) Angular Velocities

• **Reference Trajectory** Discrete bank angle commands.
Flow Diagram of Control Architecture

Feedback Loop

SAMI → Fault Tolerant Control Allocation

u_{cal} → u_{app} → Plant

Tracking Error

Reference Trajectory

Update Loop
Structured Model Reference
Adaptive Control
Akella, Schaub, Junkins (Texas A&M)

Dynamics
2\textsuperscript{nd} order differential equations

Exact kinematic relationship between position and velocity

Acceleration level relationships between forces and system parameters

\[
\begin{align*}
\dot{x} &= v \\
F &= ma \\
\dot{v} &= a = F / m
\end{align*}
\]


Structured Adaptive Model Inversion

Subbarao, Junkins (Texas A&M)

• Features
  - Dynamic inversion inner-loop, with an MRAC outer-loop to handle system uncertainties.
  - Controls are solved for explicitly:
    \[
    \text{System Model } \dot{x} = Af(x) + Bu \\
    \text{Reference Trajectory } x_r, \dot{x}_r \\
    \text{Control Law } u = B^{-1}(\dot{x}_r - Af(x) - \lambda e) \text{ so that the error dynamics becomes } \dot{e} = -\lambda e
    \]
  - Undesirable dynamics are cancelled and replaced with user specified desired dynamics.
  - Easily applicable to nonlinear systems.
  - Error dynamics can be specified.
  - Shown to be very effective for a wide variety of systems.
SAMI

• Acceleration level equation of plant rewritten

\[ I_a^* (\sigma) \ddot{\sigma} + C_a^* (\sigma, \dot{\sigma}) \dot{\sigma} = P_a^T (\sigma)(u + M_{aero}) \]

where

\[ P_a (\sigma) = J_a^{-1} (\sigma) \]
\[ I_a^* (\sigma) = P_a^T IP_a \]
\[ C_a^* (\sigma, \dot{\sigma}) = I_a^* \dot{J}_a P_a + P_a^T [P_a \dot{\sigma}] IP_a \]
\[ M_{aero} = \text{Aerodynamic Moments} \]
Aerodynamic Moments

\[ M_{aero} = \frac{1}{2} S_{ref} l_{ref} V^2 \begin{bmatrix} \beta & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \rho c_{l\beta} \\ \rho c_{m\alpha} \\ \rho c_{n\beta} \end{bmatrix} \]

\[ d^* = D_{est} d \]

\[ M_{aero} = \frac{1}{2} S_{ref} l_{ref} V^2 \begin{bmatrix} \beta & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \]
Minimal Parameterization of the Inertia Matrix

\[
\begin{bmatrix}
I_{11} & I_{12} & I_{13} \\
I_{12} & I_{22} & I_{23} \\
I_{13} & I_{23} & I_{33}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
= 
\begin{bmatrix}
a_1 & 0 & 0 & a_2 & a_3 & 0 \\
0 & a_2 & 0 & a_1 & 0 & a_3 \\
0 & 0 & a_3 & 0 & a_1 & a_2
\end{bmatrix}
\begin{bmatrix}
I_{11} \\
I_{22} \\
I_{33} \\
I_{12} \\
I_{13} \\
I_{23}
\end{bmatrix}
\]

Inertia Matrix
9 parameters

\[Ia = \Lambda(a)\theta, \quad \forall a \in \mathbb{R}^3\]

Inertia Vector
6 minimal parameters
Equations of motion can be converted to

\[ I_a^*(\sigma)\ddot{\sigma} + C_a^*(\sigma, \dot{\sigma})\dot{\sigma} = Y(\sigma, \dot{\sigma}, \ddot{\sigma})\theta \]

Tracking error

\[ \varepsilon = \sigma - \sigma_r \]

Control Law

\[ u = P_a^{-T}\{Y_a(\sigma, \dot{\sigma}, \dot{\sigma}_r, \ddot{\sigma}_r)\theta - C_{da}\dot{\varepsilon} - k_{da}\varepsilon - k_i\int \varepsilon dt\} - M_{aero} \]
Control Law

- For unknown parameters

\[
u = P_a^{-T} \{ Y_a(\sigma, \dot{\sigma}, \dot{\sigma}_r, \ddot{\sigma}_r) \hat{\theta} - C_{da} \varepsilon - k_{da} \varepsilon - k_i \int \varepsilon dt \} - \frac{1}{2} S_{\text{ref}} l_{\text{ref}} V^2 \begin{bmatrix} \beta & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \hat{d}_1 \\ \hat{d}_2 \\ \hat{d}_3 \end{bmatrix}
\]

Let
\[
\tilde{\theta} = \theta - \hat{\theta} \\
\tilde{d} = d - \hat{d}
\]

Error dynamics
\[
I_a \ddot{\varepsilon} + (C_{da} \dot{\varepsilon} + k_d \varepsilon + k_i \int \varepsilon dt) = \Psi \Phi
\]
\[
\Psi = [Y_a - P^T L] \quad \Phi = [\tilde{\theta} \quad \tilde{d}]
\]
Adaptive Laws

Let

\[ y = \begin{bmatrix} \int \varepsilon dt, \varepsilon, \dot{\varepsilon} \end{bmatrix} \]

Lyapunov Candidate

\[ V = \frac{1}{2} y^T P y + \frac{1}{2} \Phi^T P \Phi \]

• Parameter update

\[ \dot{\Phi} = \Gamma \Psi^T P y \]

• Stability analysis shows that \( \sigma \to \sigma_r \) and \( \omega \to \omega_r \) as \( t \to \infty \)
Control Allocation

• Why Control Allocation?
  ▪ More jets than desired moments
  ▪ To provide consistent and unique solution to the problem
• Implemented using Mixed Integer Linear Programming (MILP)
• Fault Tolerant Algorithm
• RCS control allocator acts as quantization element
RCS Control Allocation
Doman, Gamble and Ngo

An RCS control allocation formulation:

$$\min_u \sum_{i=1}^{3} |\tau_{i_{des}} - \sum_{k=1}^{p} T_{i,k} u_k| + \sum_{k=1}^{p} w_k u_k$$

- Desired Torque to follow reference trajectory
- Calculated using Adaptive Control

Torque provided by reaction jets

On/off Value i.e. 0 or 1
Stability Analysis

For nonlinear system

\[ \dot{x} = f(x) + G(x)u \]

- calculated control \( u = k(x) \)
- applied control \( u = q_\mu(k(x)) \)

Quantization variable (fixed)

Liberzon has proved that solutions starting in \( R_1(\mu) \) enters \( R_2(\mu) \) in finite time
Controller Evaluation

- Uncertainties in moment of mass, inertia and aerodynamic coefficients
- Initial condition errors
- Test cases shown
  - Case 1: Jets 1, 2, 17, 18 are inoperable i.e. always off
  - Case 2: Jets 2, 3, 4, 5, 8, 9, 18 are always on
  - Case 3: Torque producing capability decreased
Results

Case 1: RCS Jets

Jet 1
Jet 2
Jet 3
Jet 4
Jet 5
Jet 6
Jet 7
Jet 8
Jet 9

Time (sec)
Case 1: Bank Angle Profile
(4 jets off)
Case 3: Bank Angle Response to Failure

Introduction

(4 jets off)
Case 1: Angular Velocities
(4 jets off)
Case 1: MRPs
(4 jets off)
Case 1: Control Effort
(4 jets off)
Case 1: Commanded Moments

(4 jets off)

![Graphs showing commanded moments for different axes with time in seconds on the x-axis and moments in Newton-meters on the y-axis. The graphs compare nominal and failed jets conditions.]
Case 1: Applied Moments
(4 jets off)
Case 1: Adaptive Parameters
(4 jets off)
Case 1: Adaptive Parameters

(4 jets off)
Case 1: Altitude v/s Downrange
(4 jets off)
Case 1: Translational States
(4 jets off)
Case 2: RCS Jets
Case 2: Bank Angle Profile
(7 jets on)

Bank Angle (degrees)

Bank Angle Error (degrees)
Case 3: Bank Angle Response to Failure

Introduction
(7 jets on)
Case 2: Control Effort
(7 jets on)
Case 2: Adaptive Parameters
(7 jets on)

- $I_{xx}$
- $I_{yy}$
- $I_{zz}$

Time (sec)
Case 3: Bank Angle Profile (Reduced Torque)
Case 3: Bank Angle Response to Failure

Introduction

(Reduced Torque)
Case 3: Adaptive Parameters (Reduced Torque)

The diagrams show the time evolution of the moments of inertia $I_{xx}$, $I_{yy}$, and $I_{zz}$ with respect to time (sec) for the nominal and failed jets conditions. The $I_{xx}$ moment shows a significant response to the reduced torque, while $I_{yy}$ and $I_{zz}$ remain relatively stable.

The $I_{xx}$ moment begins at a nominal value and decreases significantly after the initial time, indicating a reduction in torque. The $I_{yy}$ and $I_{zz}$ moments, however, remain close to their nominal values throughout the time period shown, suggesting that these moments are less affected by the reduced torque.

The graphs illustrate the impact of adaptive parameters in maintaining system stability despite torque reductions.
Conclusions

- SAMI based update laws perform very well to handle the plant parameter uncertainties
- Controller provides good trajectory tracking performance even in case of failures
- Control algorithm is capable of handling different kinds of failures
Future Work

• Integration of adaptive trajectory generation
• Some algorithm is required to detect failure