Structured Adaptive Model Inversion Controller for Mars Atmospheric Flight

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Overview

- Research Motivation
- Objectives and Scope
- Mars Entry
- Vehicle Model
- Entry Simulation
- Adaptive Control
- Results
- Conclusions
- Recommendations
Motivation

Controlled entry through an uncertain environment is challenging, and adaptive guidance and control systems are a potential solution.

<table>
<thead>
<tr>
<th></th>
<th>Past</th>
<th>Future</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mission</strong></td>
<td>Robotic</td>
<td>Manned</td>
</tr>
<tr>
<td><strong>Vehicle</strong></td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td><strong>Landing Requirements</strong></td>
<td>&lt; 100 km from target</td>
<td>&lt; 5 km from target</td>
</tr>
<tr>
<td><strong>Entry Trajectory</strong></td>
<td>Ballistic</td>
<td>Controlled</td>
</tr>
</tbody>
</table>

- Controlled entry through an uncertain environment is challenging.
- Adaptive guidance and control systems are a potential solution.
Objective & Scope

- **Primary Objective:**
  - Evaluate feasibility of using adaptive control for a Mars entry vehicle under the presence of atmospheric and vehicle property uncertainties

- **Scope:**
  - Vehicle must follow a pre-computed trajectory from entry interface at 125 km through parachute deployment at approximately 12 km above the surface
  - No active guidance
Contributions

- Applied adaptive control to a Mars entry vehicle
- Successful adaptation for atmospheric parameter uncertainties
- Successful tracking large maneuvers as part of a smooth entry trajectory

NASA - JSC interested in adaptive controllers for entry vehicles.
Mars Entry - Trajectory

500km - Circular Orbit

~125 km

Atmospheric Flight

~12 km

Parachute Phase

~5 km
Mars Entry - GN&C

- Guidance and Control for Mars Entry are “decoupled” if the vehicle is trimmed
- Guidance generates a trajectory in real time
- Optimization problem to maximize final altitude with path constraints on heat rates, g-loads, dynamic pressure
- Outputs an optimal bank angle profile
- The control system tracks the bank angle profile and keeps vehicle at its trim angle-of-attack using reaction control system jets
Mars Atmosphere

- Dense enough to produce significant heat rates
- Not dense enough to decelerate vehicle
- Surface pressure at Mars < 1% sea level pressure on Earth

- Large Temperature gradients
- Large Density variations
- Mars-GRAM is used
Mars Ellipsled

- Mars Combo Lander Design Study\(^1\) at JSC concluded:
  - Ellipsled can weigh up to 78 mt, be 20 m long, and fit on current launch vehicles
  - The aeroshell is half of the launch shroud on Earth
  - Sample mission: 2 ellipsleds carrying crew, life support systems, and science experiments
  - Sufficient flight mechanics margins for human missions
  - Study used scaled-down model of 3000 kg and 3.75 m in diameter
Ellipsled - Controls

- 18 Reaction Control System (RCS) jets

<table>
<thead>
<tr>
<th>Axis</th>
<th>Ang. Rates (deg/s)</th>
<th>Moment (Nm)</th>
<th>Moment Arm (m)</th>
<th>Thrust (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll jet couple</td>
<td>2.87</td>
<td>105</td>
<td>1.88</td>
<td>28</td>
</tr>
<tr>
<td>Pitch jet couple</td>
<td>2.00</td>
<td>166</td>
<td>2.98</td>
<td>28</td>
</tr>
<tr>
<td>Yaw single jet</td>
<td>1.94</td>
<td>284</td>
<td>1.88</td>
<td>95</td>
</tr>
</tbody>
</table>
Simulation

- Nonlinear, 5DOF: 3 rotational, altitude, and downrange
- Atmosphere model: Mars-GRAM
- Vehicle model from JSC study\(^1\):
  - Rigid body, \(x_{cg}\) and \(z_{cg}\) shift for roll maneuverability, stable in pitch, trimmed at angle-of-attack = 55°
  - Trim assumption allows decoupling of guidance and control equations
- Guidance: translational equations for a point mass
- Control: rotational equations for a rigid body
Translational Equations

- **External Forces**

\[ [F]_w = \begin{bmatrix} -D - mg \sin \gamma \\ mg \cos \gamma \sin \phi \\ -L + mg \cos \gamma \cos \phi \end{bmatrix} \]

\[ [a]_w = \begin{bmatrix} \dot{v} \\ -v \dot{\gamma} \sin \phi \\ -v \dot{\gamma} \cos \phi \end{bmatrix} \]

\[ \begin{align*}
\dot{\gamma} &= \frac{L}{mv} \cos \phi - \frac{g}{v} \cos \gamma \\
\dot{v} &= -\frac{D}{m} - g \sin \gamma \\
\dot{x} &= v \cos \gamma \\
\dot{h} &= -v \sin \gamma
\end{align*} \]
Rotational Equations

- Euler’s equations:

\[
\begin{align*}
L &= \bar{q} S_{\text{ref}} L_{\text{ref}} c_{l\beta} \beta \\
M &= \bar{q} S_{\text{ref}} L_{\text{ref}} c_{m\alpha} \alpha \\
N &= \bar{q} S_{\text{ref}} L_{\text{ref}} c_{n\beta} \beta
\end{align*}
\]

where \( f \) is the sum of aerodynamic moments and control input.

- Kinematics:

\[
[w_{b/i}]_b = [w_{b/w}]_b + [w_{w/i}]_b
\]

\[
\begin{bmatrix}
p \\ q \\ r
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta \cos \phi + \sin \alpha \sin \phi & \sin \alpha \\
\sin \beta & \cos \beta \cos \phi & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta \sin \phi - \cos \alpha \sin \phi & -\cos \alpha
\end{bmatrix}
\begin{bmatrix}
\dot{\phi} \\ \dot{\gamma} \\ \dot{\beta}
\end{bmatrix}
= A^{-1}
\begin{bmatrix}
p \\ q \\ r
\end{bmatrix}
\]
Entry Trajectory

- Smooth bank angle profile to satisfy SAMI requirements

- Reference attitude must be differentiable for control law development

5th order polynomial fit for each bank angle command
Adaptive Control: MRAC

- Problem: Design controller for system with uncertainties
- Solution: Have system track the output of a reference model
- Error convergence is achieved
- Adaptive parameter convergence is not necessarily achieved

Controller:
\[ u = \tilde{a}_r \omega_r + \tilde{a}_w \omega \]

Adaptive laws:
\[ \dot{\tilde{a}}_r = -\Gamma e \omega_{\text{ref}} \]
\[ \dot{\tilde{a}}_w = -\Gamma e \omega \]

Error:
\[ e = \omega - \omega_r \]
SAMI: background

- Based on Structured MRAC and dynamic inversion
- Controller replaces undesired dynamics
- Requires exact knowledge of dynamical structure
- Adaptation mechanism compensates for the lack of knowledge of vehicle and environment properties
- For this problem, the adaptive parameters are:
  - Atmospheric density
  - Vehicle inertias
  - Aerodynamic coefficients
Adaptive Control: SAMI

- **Plant:**
  - Nonlinear in states, affine in control, uncertain parameters appear linearly.

- **Control:**
  - Dynamic Inversion and Sliding Mode Control.
  - Dynamic Inversion requires knowledge of system parameters, which are inherently uncertain.

- **Adaptive Learning Parameters:**
  - Updated in real-time, and used for the Dynamic Inversion

- **Adaptation Mechanism:**
  - Driven by error between the actual plant trajectory and the reference trajectory

- **Stability Analysis:** Guarantees plant trajectory asymptotically converges to reference trajectory in presence of parametric uncertainties and I.C. errors.
Dynamics

$2^{nd}$ order differential equations

$\ddot{x} = v$

$\dot{v} = a = \frac{F}{m}$

Exact kinematic relationship between position and velocity

Acceleration level relationships between forces and system parameters
SAM1: density adaptation

- Rotational kinematics and dynamics

\[ \dot{q} = A(q) \omega \quad \dot{q} = f(q, \omega) \]
\[ I\ddot{\omega} = -\omega \times I\omega + u \quad \dot{\omega} = g(q, \omega, p) + h(q, \omega, p)u + H(q, \omega) \]

- Adapting for density and aerodynamic coefficients

\[ g = \frac{1}{2} \rho v^2 S_{ref} l_{ref} \begin{pmatrix} \frac{c_{l}\beta}{I_x} \\ \frac{c_{m}\alpha}{I_y} \\ \frac{c_{n}\beta}{I_z} \end{pmatrix} = \frac{1}{2} v^2 S_{ref} l_{ref} [I]^{-1} \begin{pmatrix} \beta & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{pmatrix} \rho \begin{pmatrix} c_{l}\beta \\ c_{m}\alpha \\ c_{n}\beta \end{pmatrix} \]

- Separate knowns and unknowns

\[ \rho = f(h)^T \Theta \quad \text{where} \quad f(h) = [1 \ h \ h^2 \ h^3 \ h^4 \ ...]^T \]
\[ \Theta = [\Theta_0 \ \Theta_1 \ \Theta_2 \ \Theta_3 \ \Theta_4 \ ...]^T \]
\[ g = \frac{1}{2} v^2 S_{ref} l_{ref} [I]^{-1} \begin{pmatrix} \beta & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{pmatrix} f^T(H) \Theta = GL \quad \text{where} \]

G: known
L: unknown
SAMI: control law

- Enforced error dynamics
  \[ \ddot{e} + C\dot{e} + Ke = 0 \quad \text{where} \quad e = q - q_r \]

- Substituting kinematic and dynamic equations
  \[
  u = -[I] \left[ GL + \frac{\partial f^{-1}}{\partial \omega} \left( \frac{\partial f}{\partial \omega} H + \frac{\partial f}{\partial q} \dot{q} - \ddot{q}_r + C(\dot{q} - \dot{q}_r) + K(q - q_r) \right) \right]
  \]
  \[ u = -[I](GL + \Psi) \]

- Lyapunov function
  \[
  V = \frac{1}{2} \ddot{e}^T \ddot{e} + \frac{1}{2} e^T Ke + \frac{1}{2} \dot{\tilde{L}}^T \Gamma_1^{-1} \dot{\tilde{L}} + \frac{1}{2} Tr[\tilde{I}^{-T} \Gamma_2^{-1} \tilde{I}^{-1}] \]

- 
  \[
  \dot{\tilde{L}} = -\Gamma_1 G^T \dot{\tilde{e}} \]
  \[
  \dot{\tilde{I}} = -\Gamma_2 ue\dot{e}^T \]

where \( \Gamma_1 \) and \( \Gamma_2 \) are tuning parameters
SAMI: tuning

- SAMI learning rates are highly sensitive
  - Must be slow enough to keep oscillations low
  - Must be fast enough to adapt to the rapidly changing parameters
- Learning rates for atmospheric density are high
- Learning rates for inertia values are low
- Appropriate combination of learning rates for all unknown parameters was difficult to find
## MRAC vs. SAMI

<table>
<thead>
<tr>
<th></th>
<th>MRAC</th>
<th>SAMI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Error</strong></td>
<td>$e = \omega - \omega_r$</td>
<td>$e = q - q_r$</td>
</tr>
<tr>
<td><strong>Control law</strong></td>
<td>$u = \tilde{a}<em>r \omega_r + \tilde{a}</em>\omega \omega$</td>
<td>$u = -[I](GL + \Psi)$</td>
</tr>
<tr>
<td><strong>Adaptive Parameters</strong></td>
<td>Controller gains</td>
<td>Inertias, atmospheric density, and aerodynamic coefficients</td>
</tr>
<tr>
<td><strong>Tuning</strong></td>
<td>Easy*</td>
<td>Difficult</td>
</tr>
</tbody>
</table>
Controller evaluation

- What do we want to know?
  - Limitations of adaptive control for Mars entry
  - Response to rapid changes in atmosphere conditions
  - Response to initial errors
  - Limits on bank angle commands and time of maneuver

- Test Plan
  - Step response for each controller
  - Entry trajectory performance for each controller
MRAC: max bank angle

- $170^\circ$ bank command in 10 seconds
SAMI: max bank angle

- 170° bank command in 12 seconds
- Uncertainties in density, aerodynamic coefficients, and inertias
MRAC vs. SAMI: step

- SAMI has better tracking performance
- MRAC requires less control input
SAMI: density uncertainty

- $150^\circ$ bank command in 10 seconds
- $u$ with true parameters vs. $u$ with $L$ estimates
SAMI: inertia uncertainty

$u$ with true $I$ vs. $u$ with $I$ estimates
SAMI: density & inertias

- Effect of uncertainties in atmospheric density, aerodynamic coefficients, and inertias
MRAC: entry trajectory

- Steady state bank angle error
- Difficult to choose learning rates for an entire trajectory
SAMI: entry trajectory - 1

- Successful smooth trajectory tracking
- Drastic bank reversals
SAMI: entry trajectory - 2

- Inertial velocity, downrange, altitude, and flight path angle
MRAC vs. SAMI: control

- Control Energy and Control Effort

\[ u^T u \]
\[ \int_{t_0}^{t_f} u^T u \, dt \]
Conclusions

- For Mars entry:
  
  adaptive controller can track smooth commands up to 170° in 12 seconds

  learning rates tunable for tracking entire entry trajectory

- SAMI provided better tracking performance than MRAC, but uses more control

- MRAC easier to design than SAMI:
  
  MRAC gains easy to tune

  SAMI gains difficult to tune

  highly sensitive to changes in atmospheric density and vehicle properties

- SAMI is a candidate solution for the Mars entry control problem
Next Steps

- Adapt for uncertainties in control effectiveness
- Introduce control allocation
- Add additional controlled variables:
  - lift
  - angle-of-attack
- Include adaptive guidance
- Sampled-data and L1 adaptive controllers
- Compare adaptive controller to phase plane controller
- Investigate certification issues for adaptive
Acknowledgements

- National Science Foundation Graduate Research Fellowship
- EG Division, NASA Johnson Space Center
- NASA civil servants Alan Strahan, Mark Hammerschmidt, David Kanipe
- Dr. Bill Schneider, NASA Johnson Space Center (ret)
Questions?
Comparison

- **Structured Adaptive Inversion Controller (SAMI)**
  - Operates linearly on nonlinearities in model
  - Starts with best dynamical model
  - Adaptively tunes affine parameters that are physically modeled

- **Intelligent Adaptive Control**
  - Neural network basis functions as acceleration errors in the state model
  - Adapts weights on neural net and B matrix

- **Approaches are Equal**
  - If SAMI uses additional affine parameters as perturbations on the basis functions
Ellipsled Model Properties

- L/D ~ .46 to .52
- Diameter = 3.75 m
- Length = 6.323 m
- S ref = 11.045 m²
- Z cg = -.175m
- X cg = 52.88%
- Mass = 3000 kg
- Ixx = 2983 kg·m²
- Iyy = 4909 kg·m²
- Izz = 5683 kg·m²

<table>
<thead>
<tr>
<th>α</th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$C_{n\alpha}$</th>
<th>$C_{max}$</th>
<th>$C_{l\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>0.6525</td>
<td>1.5683</td>
<td>-0.02</td>
<td>0.037</td>
<td>-2.279</td>
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<tr>
<td>50°</td>
<td>0.6595</td>
<td>1.7407</td>
<td>0</td>
<td>0.019</td>
<td>-2.354</td>
</tr>
<tr>
<td>55°</td>
<td>0.6335</td>
<td>1.9107</td>
<td>0.015</td>
<td>0.000</td>
<td>-2.414</td>
</tr>
<tr>
<td>60°</td>
<td>0.5738</td>
<td>2.0695</td>
<td>0.034</td>
<td>-0.211</td>
<td>-2.462</td>
</tr>
<tr>
<td>65°</td>
<td>0.4818</td>
<td>2.2083</td>
<td>0.05</td>
<td>-0.428</td>
<td>-2.495</td>
</tr>
</tbody>
</table>

Trim angle-of-attack = 55 deg
Previous Control System

- Derived from the Shuttle FCS
- Fixed deadbands for errors and for RCS signals that feed into the jet hysteresis functions

\[ Y_{RCS} = K_\alpha (\alpha_c - \alpha) + K_q q \]
\[ X_{RCS} = K_{X\phi} (\phi_c - \phi) + K_{X_{Pstab}} P_{stab} + K_{X_{Rstab}} R_{stab} + K_{X_\beta} \beta \]
\[ Z_{RCS} = K_{Z\phi} (\phi_c - \phi) + K_{Z_{Pstab}} P_{stab} + K_{Z_{Rstab}} R_{stab} + K_{Z_\beta} \beta + K_z \int_{\text{TrimIntegral}} \]

- Pstab is equivalent to bank about the velocity vector and Rstab is equivalent to beta dot
Rotational Kinematics

- Body angular velocity in body frame

\[
[w_{b/i}]_b = [w_{b/w}]_b + [w_{w/i}]_b
\]

\[
[w_{b/i}]_b = [C^b_w][[w_{b/w}]_w + [w_{w/i}]_w]
\]

where:

\[
[C^b_w] = [C_2(\alpha)] [C_3(-\beta)] = \begin{bmatrix}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha
\end{bmatrix}
\]

\[
[w_{w/i}]_w = \begin{bmatrix}
-\dot{\xi} \sin \gamma + \dot{\phi} \\
\dot{\xi} \cos \gamma \sin \phi + \dot{\gamma} \cos \phi \\
\dot{\xi} \cos \gamma \cos \phi + \dot{\gamma} \sin \phi
\end{bmatrix}
\]

\[
[w_{b/w}]_w = \begin{bmatrix}
0 \\
0 \\
-\dot{\beta}
\end{bmatrix}
\]

- Kinematic equations:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\gamma} \\
\dot{\beta}
\end{bmatrix} = \begin{bmatrix}
cos \alpha \cos \beta & sin \beta & sin \alpha \cos \beta \\
-sin \alpha \cos \beta / cos \phi & cos \beta / cos \phi & -sin \alpha sin \beta / cos \phi \\
-cos \alpha sin \beta tan \phi + sin \alpha & -cos \beta tan \phi & sin \alpha sin \beta tan \phi - cos \alpha
\end{bmatrix} \begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]
References

1. Ellipsled Report from JSC


3. K. Subbarao, J. Junkins. SAMI Applied to Tracking Aggressive Aircraft Maneuvers. AIAA 2001-4019