Preliminary Results Of Vehicle Formation Control Using Dynamic Inversion

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Overview

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- One-dimensional particle formation control
  - Development of equations of motion
  - Dynamic inversion control law
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Motivation

- We wish to study formation flight of air vehicles.
- Why Fly In Formation?
  - Strategic Military Reasons.
    » Airplanes in combat usually don’t fly alone.
    » Multiple aircraft can be used for cooperative strikes.
  - Drag Reduction.
    » Flying in the “sweet spot” has been shown to reduce drag due
to tilt of the lift vector of wing in the influence of the vortex.
  - Difficult to do.
  - Applications to any kind of aircraft.

NASA Dryden Aircraft In Formation Flight
Motivation (II)

- Drag Reduction.
  » Flying in the “sweet spot” has been shown to reduce drag due to tilt of the lift vector of wing in the influence of the vortex.
  » Problem: flying in the vortex is difficult.

NASA Dryden Aircraft In Formation Flight
Work on this subject has included:

- UCLA (Late 80’s and Early 1990’s) – Automated Highway Systems for cars

  » First-cut designs were leader-follower

  - Formations were look-ahead only
  - Important to understanding string stability

- Early 1990’s – Air Force Research Lab (AFRL) began efforts to develop control algorithms for aircraft

  » Pachter et al. worked on mostly linear controllers, gradually increasing complexity of controller
History (II)

– Mid 1990’s – (Speyer, Innocenti, and others).
  » Mostly linear controllers developed.
  » Emphasis on decentralized controllers.
    ■ Innocenti uses imaginary point as guide to the formation.
  » Aerodynamic interaction forces considered.
– 2000 – 2001 (AFRL (Singh), Lavretzky).
  » Adaptive controllers to handle uncertain aerodynamics of vortex.
  » Theoretical framework for two-dimensional control of vehicles using an imaginary point as reference.

■ Common Feature: Most work is leader-follower based, decentralized control, and much emphasis is placed on aerodynamic interaction.
Research Objective

- Develop and study alternative control algorithms that can be implemented in aircraft formation flight.
  - Stable.
  - Reduce information requirements.
  - Improve redundancy and fault-tolerance of the system.
- Why airplanes?
  - Because that’s our area of expertise.
  - Algorithms are developed for aircraft but basic principles are intended to be applied to other kinds of vehicles.
One-dimensional Formation Control
A Mechanical Structure Analogy

- Original Concept By Dr. Junkins
  - “Let’s connect a series of point masses by springs and dampers, develop equations of motion, and see what happens”

- We will not show the derivation of the equations of motion for this, because it is very similar to the one we will show next
One-dimensional Control Using Dynamic Inversion

The basic coordinate system for the development of equations of motion is:
Equations of Motion

- Define position and control vectors:

\[ \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix}^T \quad \mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & \cdots & u_n \end{bmatrix}^T \]

- Define new set of generalized coordinates \( \mathbf{q} \):

\[ \mathbf{q} = \begin{bmatrix} x_c & x_{12} & x_{23} & \cdots & x_{n-1,n} \end{bmatrix}^T \]

\[ x_{k,k+1} = x_{k+1} - x_k \]

\[ x_c = \frac{1}{m} \sum_{i=1}^{n} m_i x_i \quad \text{where} \quad m = \sum_{i=1}^{n} m_i \]
Equations of Motion (II)

- Developed using LaGrange’s Equations:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = Q
\]

- Kinetic energy (T) is:

\[
T = \frac{1}{2} \dot{x}^T \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & \ddots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & m_n \end{bmatrix} \dot{x} = \frac{1}{2} \dot{q}^T \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & \ddots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & m_n \end{bmatrix} R \dot{q}
\]

\[
T = \frac{1}{2} \dot{q}^T \tilde{M} \dot{q}
\]

\(\tilde{M}\) is both positive definite and symmetric.
Equations of Motion (III)

- To find generalized force vector, $\mathbf{Q}$, we use a virtual work approach:

$$\delta W = \sum_{i=1}^{n} u_i \delta x_i = \mathbf{u}^T \delta \mathbf{x}$$

$$\delta W = \mathbf{u}^T \delta \mathbf{x} = \mathbf{u}^T \delta (\mathbf{Rq}) = \mathbf{u}^T \mathbf{R} \delta \mathbf{q}$$

- We compare this result with:

$$\delta W = \mathbf{Q}^T \delta \mathbf{q}$$

- And conclude:

$$\mathbf{Q} = \mathbf{R}^T \mathbf{u}$$
The final equation of motion becomes:

\[
\frac{d}{dt} \left( \frac{1}{2} \hat{q}^T \tilde{M} \hat{q} \right) - \frac{\partial}{\partial \hat{q}} \left( \frac{1}{2} \hat{q}^T \tilde{M} \hat{q} \right) = R^T u
\]

\( \tilde{M} \ddot{q} = R^T u \)
Dynamic Inversion Control Law

Define the error, \( z \):
\[
\begin{align*}
  z &= q - q_{\text{des}} \\
  \dot{z} &= \dot{q} + \dot{q}_{\text{des}}
\end{align*}
\]

Substitute back into our equation of motion:
\[
\begin{align*}
  \tilde{M}(\dot{z} + \dot{q}_{\text{des}}) &= R^T u \\
  \tilde{M} \ddot{z} &= R^T u - \tilde{M} \dot{q}_{\text{des}}
\end{align*}
\]

Choose \( u \) to yield desired error dynamics:
\[
\begin{align*}
  u &= (R^T)^{-1}(\tilde{M}q_{\text{des}} - \tilde{C}z - \tilde{K}z)
\end{align*}
\]

\( \tilde{K} \) and \( \tilde{C} \) are positive diagonal matrices

And our closed-loop error dynamics become:
\[
\tilde{M} \ddot{z} + \tilde{C}z + \tilde{K}z = 0
\]
Lyapunov Stability Analysis

- We choose the following Lyapunov function:

\[ \overline{z} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \quad V(\overline{z}) = \frac{1}{2} \dot{z}^T \begin{bmatrix} K & 0 \\ 0 & \tilde{M} \end{bmatrix} \overline{z} = \frac{1}{2} \dot{z}^T \tilde{M} \dot{z} + \frac{1}{2} z^T \tilde{K} z \]

- V is positive definite

- Time Derivative of V:

\[ \dot{V}(\overline{z}) = \dot{z}^T \tilde{M} \dot{z} + \dot{z}^T \tilde{K} z \]

\[ \dot{V}(\overline{z}) = -\dot{z}^T (\tilde{C} \dot{z} + \tilde{K} z) + \dot{z}^T \tilde{K} z \]

\[ \dot{V}(\overline{z}) = -\dot{z}^T \tilde{C} \dot{z} \]
Lyapunov Stability Analysis (III)

- Minor Problem:
  - The time derivative of $V$ is negative semi-definite
    » It can be zero if $\dot{z}$ is zero and $z$ is some non-zero value, which is NOT an equilibrium point
  - We can deal with this problem using LaSalle’s theorem, but we’ll use Junkins and Kim’s approach, as it is easier to understand:
    » Show that the first non-zero odd derivative in the set where $\ddot{V}$ vanishes is negative definite
    » Second Derivative:
      $$\ddot{V}(\bar{z}) = -2\bar{z}^T \bar{C} \bar{z}$$
      - This one vanishes as well
Lyapunov Stability Analysis (III)

» Third time derivative:

\[ \dddot{V}(\bar{z}) = -2(z^T \tilde{K}^T \tilde{M}^{-1})^T \tilde{C}\tilde{M}^{-1}\tilde{K}z \]

\[ \dddot{V}(\bar{z}) = -2(s^T \tilde{C}s), \text{ where } s = \tilde{M}^{-1}\tilde{K}z \]

- This is negative definite in the set of interest.

- We have shown that the error dynamics are asymptotically stable.
Lyapunov Stability Analysis (IV)

- Global Stability:
  - Error dynamics globally stable if \( V(\bar{z}) \) approaches infinity as \( \|\bar{z}\| \) approaches infinity.
  - Redefine the Lyapunov Function:
    \[
    V(\bar{z}) = \frac{1}{2} \bar{z}^T \begin{bmatrix} \tilde{K} & 0 \\ 0 & \tilde{M} \end{bmatrix} \bar{z} = \frac{1}{2} \bar{z}^T \tilde{K} \bar{z} + \frac{1}{2} \bar{z}^T \tilde{M} \bar{z}
    \]
  - Implementing the transformation \( \bar{z} = Py \), where \( P \) is the matrix of normalized eigenvectors of \( A \), the following results are seen:
    » \( \bar{z} \) is of the order of \( y \)
    » \( P^T AP \) yields a diagonal matrix with the eigenvalues of \( A \)
Lyapunov Stability Analysis (V)

» The Lyapunov function can be expressed as:

\[
V(\mathbf{z}) = \frac{1}{2} \mathbf{y}^T \begin{bmatrix}
\lambda_1 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \lambda_{2n}
\end{bmatrix} \mathbf{y} = \frac{1}{2} \sum_{i=1}^{2n} \lambda_i y_i^2
\]

» It is obvious that the Lyapunov function \( V \) approaches infinity as \( y \) approaches infinity. Since \( y \) is of the same order as \( \mathbf{z} \), then \( V \) approaches infinity as \( \|\mathbf{z}\| \) approaches infinity

■ Global asymptotical stability is finally shown
The Dynamic inversion control law has the following form:

$$u = (R^T)^{-1}(\ddot{M}q_{des} - \ddot{C}z - \ddot{K}z)$$

- Also, note:

$$(R^T)^{-1} = \begin{bmatrix} m_1/m & -1 & 0 & 0 & \ldots & 0 \\ m_2/m & 1 & -1 & 0 & \ldots & 0 \\ m_3/m & 0 & 1 & -1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ m_n/m & 0 & 0 & 1 & \ldots & 0 \\ m/m & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
Information Requirements (II)

- Carrying out the matrix algebra, we realize:
  - Each particle must have the second derivative of the desired trajectory for all generalized coordinates. That is, the desired center of mass location and geometric spacing must be known \(a\) \(priori\).
  - Each particle must know the current location of the center of gravity of the system.
  - Each particle must know the location of the particles immediately next to it.
  - Each particle does not need to know the location of any other particles.
Information Requirements (III)

Information Flow Diagrams

[Diagram 1]

[Diagram 2]

[Diagram 3]

[Diagram 4]
Numerical Example

- Simulink® used to test the control law on a set of 5 particles
  - Particles are to follow a sinusoidal wave and maintain a desired constant spacing
  - Spacing could also be a function of time if desired
Numerical Example (II)

- This is presented to show particle motion and error behavior
Mechanical Structure Analogy vs. Dynamic Inversion

- We did not show it, but for the one-dimensional case, the structural analogy equations of motion are very similar to those derived here.
  - Intuitively, one can get a feel for why the two are similar, we are commanding a control law that behaves like a spring-damper system in a linear system.

- If dynamic inversion, then, gave us a usable control law in the one-dimensional case, would it do the same for the two-dimensional case?
Two-Dimensional Formation Control

- We define a new set of generalized coordinates
Two-Dimensional Formation Control (II)

- And we apply inertially-oriented control forces
2-D Equations of Motion

- Developed using LaGrange’s equations, just as we did before.

\[
\begin{bmatrix}
\ddot{x}_{eg} \\
\ddot{y}_{eg} \\
\ddot{d}_{12} \\
\ddot{\psi}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{m} & 0 & \frac{1}{m} & 0 & 0 \\
0 & \frac{1}{m} & 0 & \frac{1}{m} & 0 \\
-\cos(\gamma) & \sin(\gamma) & \cos(\gamma) & \sin(\gamma) & 0 \\
\sin(\gamma) & -\cos(\gamma) & -\sin(\gamma) & \cos(\gamma) & 0 \\
\cos(\psi) & -\sin(\psi) & 0 & 0 & \cos(\psi) \\
\sin(\psi) & -\cos(\psi) & 0 & 0 & \sin(\psi) \\
\end{bmatrix}
\begin{bmatrix}
U_{1x} \\
U_{1y} \\
U_{2x} \\
U_{2y} \\
U_{3x} \\
U_{3y}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\dot{\gamma}^2 d_{12} \\
-2\dot{\gamma} \dot{d}_{12} \\
\psi^2 d_{13} \\
-2\psi \dot{d}_{13}
\end{bmatrix}
\]
Dynamic Inversion Control Law

A stabilizing control law (defining \( z \) the same way we defined it before) is:

\[
\begin{bmatrix}
\frac{1}{m} & 0 & \frac{1}{m} & 0 & \frac{1}{m} & 0 \\
0 & \frac{1}{m} & 0 & \frac{1}{m} & 0 & \frac{1}{m} \\
\frac{\cos(\gamma)}{m_1} & \frac{\sin(\gamma)}{m_1} & \frac{\cos(\gamma)}{m_2} & \frac{\sin(\gamma)}{m_2} & 0 & 0 \\
\frac{\sin(\gamma)}{m_1 d_{12}} & \frac{-\cos(\gamma)}{m_1 d_{12}} & \frac{-\sin(\gamma)}{m_2 d_{12}} & \frac{\cos(\gamma)}{m_2 d_{12}} & 0 & 0 \\
\frac{-\cos(\psi)}{m_1} & \frac{-\sin(\psi)}{m_1} & 0 & 0 & \frac{-\cos(\psi)}{m_3} & \frac{-\cos(\psi)}{m_3} \\
\frac{\sin(\psi)}{m_1 d_{13}} & \frac{-\cos(\psi)}{m_1 d_{13}} & 0 & 0 & \frac{-\sin(\psi)}{m_3 d_{13}} & \frac{-\cos(\psi)}{m_3 d_{13}}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0 \\
\gamma^2 d_{12} \\
-\frac{2\gamma d_{12}}{d_{12}} \\
\psi^2 d_{13} \\
-\frac{2\psi d_{13}}{d_{13}}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\ddot{\chi}_{cgd} \\
\ddot{\gamma}_{cgd} \\
\dddot{\gamma}_{12d} \\
\ddot{\psi}_{13d} \\
\dddot{\psi}_{13d}
\end{bmatrix}
\]

\[= \mathbf{C} \mathbf{z} - \mathbf{K} \mathbf{z}\]
Dynamic Inversion Control Law (II)

- This control law yields the following closed-loop error dynamics:
  \[ \ddot{z} + \tilde{C} \dot{z} + \tilde{K} z = 0 \]
  - This is exactly the same error dynamics we studied before, but the "mass" matrix has just been replaced by the identity matrix, so our Lyapunov analysis still holds for this system.

- **BUT** there is some bad news:
  - This control law FULLY couples all states, all particles need to know all measurements at any given time.
2-D Control Law Numerical Example

- Control law was implemented for response to initial position errors only:
Since dynamic inversion doesn’t do what we want for the 2-D case, we’ll go back to inserting virtual springs and dampers into the system, for two particles, this means:
Structural Analogy 2-D Equations

- LaGrangian approach used to obtain these:

\[
\begin{bmatrix}
\frac{1}{m} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{m} & 0 & 0 & 0 \\
-\cos(\gamma) & \sin(\gamma) & \frac{1}{m} & 0 & 0 \\
\sin(\gamma) & -\cos(\gamma) & \frac{1}{m} & 0 & 0 \\
\frac{1}{m} d_{12} & \frac{1}{m} d_{12} & \frac{1}{m} d_{12} & \frac{1}{m} d_{12} & \frac{1}{m} d_{12} \\
-\sin(\gamma) & -\cos(\gamma) & 0 & 0 & \frac{1}{m} d_{12} \\
\cos(\gamma) & \sin(\gamma) & \frac{1}{m} d_{12} & \frac{1}{m} d_{12} & \frac{1}{m} d_{12} \\
\sin(\gamma) & -\cos(\gamma) & 0 & 0 & \frac{1}{m} d_{12} \\
\frac{1}{m} d_{13} & \frac{1}{m} d_{13} & \frac{1}{m} d_{13} & \frac{1}{m} d_{13} & \frac{1}{m} d_{13} \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
U_{1x} \\
U_{1y} \\
U_{2x} \\
U_{2y} \\
U_{3x} \\
U_{3y} \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Control Law Yet To Be Found

\[
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\frac{1}{m} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{m} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{m_1 + m_2}{m_1 m_2} & 0 & \cos(\gamma) & -\sin(\gamma) \\
0 & 0 & \frac{m_1 m_2}{m_1 m_2 d_{12}} & 0 & \sin(\gamma - \gamma) & \cos(\gamma) \\
0 & 0 & \frac{1}{m_1 + m_3} & 0 & \sin(\gamma - \gamma) & -\cos(\gamma) \\
0 & 0 & \frac{m_1 + m_3}{m_1 m_3 d_{13}} & 0 & \cos(\gamma) & \sin(\gamma) \\
0 & 0 & \frac{1}{m_1 m_3 d_{13}} & 0 & \cos(\gamma) & \sin(\gamma) \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
K_1 (d_{12} - d_{12}) + K_2 (\dot{d}_{12} - \ddot{d}_{12}) \\
K_3 (\gamma - \gamma_d) + K_4 (\dot{\gamma} - \dot{\gamma}_d) \\
K_5 (d_{13} - d_{13}) + K_6 (\dot{d}_{13} - \ddot{d}_{13}) \\
K_7 (\psi - \psi_d) + K_8 (\dot{\psi} - \dot{\psi}_d) \\
\end{bmatrix}
\]
Conclusions

For the one-dimensional control case, we are able to develop an effective formation flight control algorithm.

– Using the center of mass to guide the formation is a valid approach that yields achievable control laws for one-dimensional formation flight.

– The control law developed is globally asymptotically stable.

– The information requirements are not unrealistic for current aircraft capabilities.
Further Research

- Find viable control law for two-dimensional system of particles
- Implement learned principles to a system of nonholonomic vehicles (aircraft)
- Investigate/quantify string stability properties of the proposed control laws
- Incorporate aerodynamic coupling
- Develop collision avoidance/more sophisticated algorithms
Questions?