STRUCTURED ADAPTIVE MODEL INVERSION
CONTROL TO SIMULTANEOUSLY HANDLE
ACTUATOR FAILURE & ACTUATOR SATURATION

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Overview

- Problem Definition
  - Actuator Failure and Fault Tolerance.
  - Actuator Saturation.
- Introduction to basic concepts of SMRAC and SAMI.
- Mathematical development & Stability Analysis
- Nonlinear 6 dof simulation of an F-16 with thrust vectoring.
- Conclusions and Future Work.
Research Objective

Develop a Structured Adaptive Model Inversion (SAMI) Controller to:

- Accommodate actuator failures in a redundantly actuated system.
- Facilitate correct adaptation in the presence of actuator saturation limits.
Handling Actuator Failure

- **Problem:**
  - Redundantly actuated system to achieve optimality wrt the control effort.
  - Some controls fail but the system is controllable with the remaining controls.

- **Mission:**
  - To reconfigure the controller to maintain stability and performance
Various types of Actuator Failure

1. Fail hard over.
2. Fail neutral.
3. Jammed or frozen actuator: Actuator gets stuck at an arbitrary position.
4. Oscillating actuator: Smart actuator malfunctions and goes hard over in the opposite direction each time the command signal crosses a specific value. Somewhat rare, but it has occurred in operation.
5. Floating damped actuator.
6. Lost or Damaged surface.
General method to handle Actuator Failure

- Failure detection and isolation algorithm.
- Calculation of the new control effectiveness matrix.
- Controller redesign by recalculating the control gains.
- This method depends on the efficacy of the failure detection algorithm.
  - The failure may not be detected, or
  - The algorithm may give a false warning

- To remove the need for a failure detection algorithm, we will use an Direct Adaptive Controller that is constantly updating itself.
Problems in Adaptive systems due to Saturation

- Adaptation is based on the tracking error.

- Tracking error has contributions due to
  - Initial Condition Error.
  - Parametric Uncertainties in the model.
  - Control Saturation.

- We are adapting only to parametric uncertainties in the system model.
- Including the error component due to saturation will cause wrong adaptation
Usual approaches to handle Adaptation Problems due to Saturation

- Reduce the adaptation rate or completely stop adaptation when the control saturates.

- If the input saturates due to an aggressive reference trajectory, the reference command is adjusted so that the input does not saturate (Johnson and Calise).
Structured Model Reference Adaptive Control

(Akella and Junkins)

Dynamics

2nd order differential equations

\[ \dot{x} = v \]

\[ \dot{v} = a = F / m \]

Exact kinematic relationship between position and velocity

Acceleration level relationships between forces and system parameters

\[ F = ma \]
Structured Adaptive Model Inversion

(Subbarao and Junkins)

- Dynamic inversion is used to solve for controls explicitly
  
  from \( \dot{x} = Ax + Bu \)

  we can calculate the control as

  \[
  u = B^{-1}(\dot{x} - Ax)
  \]

- Adaptive control is used to estimate the parameters required in the controller.

- Controller drives the error between the states of the model and the plant to zero.

- The error dynamics can be specified
Controller to handle Actuator Freezes

- Controller freeze: The control surface remains fixed at a position which may or may not be zero.
- Mathematical model for control freeze:

\[
\mathbf{u}_{\text{applied}} = D\mathbf{u}_{\text{calculated}} + \mathbf{E}
\]

where \(D\) is a constant matrix and \(\mathbf{E}\) is a constant vector.

\[
\begin{bmatrix}
\mathbf{u}_{a_1} \\
\mathbf{u}_{a_2} \\
\mathbf{u}_{a_3}
\end{bmatrix} = \begin{bmatrix}
D_{11} & 0 & 0 \\
0 & D_{22} & 0 \\
0 & 0 & D_{33}
\end{bmatrix} \cdot \begin{bmatrix}
\mathbf{u}_{c_1} \\
\mathbf{u}_{c_2} \\
\mathbf{u}_{c_3}
\end{bmatrix} + \begin{bmatrix}
\mathbf{E}_1 \\
\mathbf{E}_2 \\
\mathbf{E}_3
\end{bmatrix}
\]
Mathematical formulation of the control law
Definition of the Structured Model

- Mathematical model of the system

\[ M(q, \dot{q})\ddot{q} = G(q, \dot{q}) + Cu \]
\[ \ddot{q} = M^{-1}G + M^{-1}Cu \]

- Structured model

\[ \dot{\sigma} = J(\sigma)w \]
\[ \dot{w} = A(\sigma, w) + B(\sigma, w)u_{app} \]
\[ \dot{w} = A(\sigma, w) + B(\sigma, w)Du_{cal} + B(\sigma, w)E \]
Definition of Reference Model & Errors

- Reference model

\[ \dot{\sigma}_r = J(\sigma_r)w_r \]
\[ \dot{w}_r = w_{\text{desired}} \]

- Error in the position and the velocity level states

\[ s = \sigma - \sigma_r \]
\[ x = w - w_r \]
Calculating control by Dynamic Inversion

- We want the error to go to zero so we specify the following dynamics:

\[ \dot{x} = A_h x + \phi \]

- Using dynamic inversion to solve for the control

\[ u_{cal} = \text{pinv}(BD)(-A - BE + \psi) \]

\[ \text{pinv}(BD) = (BD)^T (BD(BD)^T)^{-1} \]

- Where

\[ \psi \triangleq \dot{w}_r + A_h \cdot x + \phi \]
Adaptive Learning Parameters

- But actual parameters are not known, so using best estimated values
  \[ \mathbf{u}_{cal} = \text{pinv}(B_{est} D)(-C_a \mathbf{A}_{est} - B_{est} \mathbf{E} + \psi) \]

- We have introduced a parameter \( C_a \) such that
  \[ C_a^* \mathbf{A}_{est} = \mathbf{A} \]

- Let
  \[ \tilde{C}_a = C_a^* - C_a \]
  \[ \tilde{D} = D^* - D \]
  \[ \tilde{E} = E^* - E \]
Modified Reference due to Control Hedging

- Due to position limits on the control
  
  \[
  \text{if } \quad \text{abs} \left( u_{cal} \right) > u_{\text{max}} \quad \quad u_{\text{calcsat}} = u_{\text{max}} \text{sign} \left( u_{cal} \right)
  \]

- The control that could not be supplied because of saturation
  
  \[
  \delta = u_{cal} - u_{\text{calcsat}}
  \]

- The acceleration corresponding to this control is the hedging signal.

\[
B_{est} D\delta
\]
Modified Reference due to Control Hedging

\[
\dot{x} = A_h x + \varphi + \tilde{C}_a A_{est} + B_{est} \tilde{D}u_{calcsat} + B_{est} \tilde{E} - B_{est} D \delta
\]

- Subtract the hedging signal from the reference.

\[
\dot{w} - (\dot{w}_r - B_{est} \tilde{D} \delta) = A_h x + \varphi + \tilde{C}_a A_{est} + B_{est} \tilde{D}u_{calcsat} + B_{est} \tilde{E}
\]

- Modified Reference \( \dot{w}_r - B_{est} D \delta \)

- The hedging signal acts as a disturbance on the reference model

- For a feasible reference trajectory, the control becomes unsaturated and the modified reference converges to the original desired reference.
Incorporating Position in the Tracking Error

- Let the tracking error be defined as
  \[ y \triangleq \dot{s} + \lambda s \]

- For the error to converge we want
  \[ \dot{y} = A_h y \]

- Which gives the value for the forcing function \( \phi \)
  \[ \dot{x} = A_h x + \phi \]
Lyapunov Analysis

- Consider the error departure function as the lyapunov function

\[ V = y^T P y + \text{Tr}(\tilde{C}_a^T W_1 \tilde{C}_a + \tilde{D}^T W_2 \tilde{D} + \tilde{E}^T W_3 \tilde{E}) \]

- Derivative of the lyapunov function

\[ \dot{V} = -y^T Q y \]
\[ + 2\text{Tr}[\tilde{C}_a^T (J^T P y A_{est}^T + W_1 \tilde{C}_a) + \tilde{D}^T (B_{est}^T J^T P y u_{cal}^T + W_2 \tilde{D})] \]
\[ + 2\tilde{E}^T (B_{est}^T J^T P y + W_3 \tilde{E}) \]
Update Law for Adaptive Parameters from Lyapunov Analysis

- Retaining only the negative definite part and setting all other terms to 0

\[
\dot{C}_a = -W_1^{-1}(J^T PyA_{est}^T) \quad \quad \dot{C}_a^* - \dot{C}_a = -W_1^{-1}(J^T PyA_{est}^T)
\]

- But \( C_a^* \) is assumed to be constant. Hence

\[
\dot{C}_a = W_1^{-1}(J^T PyA_{est}^T)
\]

- Similarly we get update laws for \( D \) matrix and the \( E \) vector
Stability Analysis

- Lyapunov Function \( V = V(y, \tilde{C}_a, \tilde{D}, \tilde{E}) \)

- Derivative is negative semi-definite \( \dot{V} = -y^T Q y \)

- So we conclude \( y \in L_2 \cap L_\infty \) and \( \tilde{C}_a, \tilde{D}, \tilde{E} \in L_\infty \)

- By analyzing the various components of \( \dot{y} \) we can say that \( \dot{y} \in L_\infty \)
Stability Analysis

- From Barbalat’s Lemma we conclude $y \to 0$ as $t \to \infty$
  
  $$y \triangleq \dot{s} + \lambda s$$

- $y$ can be zero only if $s = 0$ and $\dot{s} = 0$

- Thus the states of the plant converge to the reference and perfect tracking can be achieved.
Numerical Example
aileron actuator failure

- Nonlinear, 6-DOF F-16 augmented with thrust vectoring.
  - Flight Condition: 0.8/50k
  - Right aileron fails at time = 5 seconds and settles to -1 degree.
    • Rolling moment counteracted by differential stabilator.
    • Other controls saturate.
- Test Cases:
  - I Adaptation stopped upon saturation
  - II Continuous adaptation
- 8 Controls
  • Differential stabilators: 2
  • Differential ailerons: 2
  • Rudder: 1
  • Thrust Vectoring: 3 (Tx, Ty, Tz)

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>Angular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>x, y, z</td>
<td>φ, θ, ψ</td>
</tr>
<tr>
<td>Velocity</td>
<td>u, v, w</td>
<td>p, q, r</td>
</tr>
</tbody>
</table>
Time Histories of the Angular states

Case I: Adaptation stopped on Saturation
Time Histories of the Linear states

Case I: Adaptation stopped on Saturation
Time Histories of the Controls

Case I: Adaptation stopped on Saturation

All control surface deflections are in degrees and all thrusts are in lbs
Time Histories of the Angular states

Case II: Adaptation during Saturation

Angular States of the Aircraft

- $\phi$ (deg)
- $\theta$ (deg)
- $\psi$ (deg)

- $p$ (deg/sec)
- $q$ (deg/sec)
- $r$ (deg/sec)

Adaptive
Reference

Time (sec)

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Time Histories of the Linear states

Case II: Adaptation during Saturation

Linear States of the Aircraft

- Adaptative
- Reference
Conclusions

- Simultaneously handles actuator failure and actuator saturation, without fault detection and isolation algorithm.

- Fault Tolerance – Add mathematical model of controller failure to plant model.

- Correct adaptation in presence of saturation - modifying the reference.

- Adaptive learning parameters may not converge.

- Global asymptotic stability of the tracking errors.
Future Work

- Actuator rate saturation.
  - Use tanh function to smooth actuator position and rate limits.

- Constrain adaptation of the learning parameters by using adaptive laws with projection.

\[
\mathbf{u}_{\text{cal}} = \text{pinv}(\mathbf{B}_{\text{est}} \mathbf{D})(-\mathbf{C}_{a} \mathbf{A}_{\text{est}} - \mathbf{B}_{\text{est}} \mathbf{E} + \mathbf{\psi})
\]
Thank You

Questions?