Structured Adaptive Model Inversion Control to Simultaneously Handle Actuator Failure and Actuator Saturation

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STRUCTURED ADAPTIVE MODEL INVERSION CONTROL TO
SIMULTANEOUSLY HANDLE ACTUATOR FAILURE AND ACTUATOR
SATURATION

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ABSTRACT
Traditional adaptive control lacks rigorous theoretical treatment for control in the presence of actuator saturation. Generally, adaptation is stopped as soon as the control saturates to avoid incorrect adaptation. Adaptation in the presence of saturation may be critical, especially when the controller is recovering from a failure. This paper presents an Adaptive Control methodology that facilitates correct adaptation in the presence of actuation saturation limits. The central idea is to modify the reference trajectory on saturation, in such a way that the modified trajectory approximates the original reference as close as possible, and can be tracked within saturation limits. Nonlinear six degree of freedom simulations of an F-16 type aircraft are shown to demonstrate this control scheme.

INTRODUCTION
In actual practice, dynamic systems that are being controlled may be poorly modelled or the parameters of the system may be varying with respect to the operating environment. To compensate for poor modelling or parameter variations, an adaptive controller is used whose parameters are updated online, based on the signals of the system.

Structured Adaptive Model Inversion (SAMI)1 is based on the concepts of Feedback Linearization,2 Dynamic Inversion, and Structured Model Reference Adaptive Control (SMRAC).3,4 In SAMI, dynamic inversion is used to solve for the control. The dynamic inversion is approximate, as the system parameters are not modelled accurately. An adaptive control structure is wrapped around the dynamic inverter to account for the uncertainties in the system parameters. This controller is designed to drive the error between the output of the actual plant and that of a model reference to zero, as in model reference adaptive control.5–7 Most dynamic systems can be broken into an exactly known kinematic level part, and a momentum level part with uncertain system parameters. The adaptation included in this framework can be limited to only the uncertain momentum level equations, to simplify the adaptation.8 The closed-loop system is shown to be globally stable for trajectories without singularities. However, the adaptively estimated parameters do not converge to the actual parameters of the system. SAMI has been shown to be effective for tracking spacecraft9 and aggressive aircraft maneuvers.10 The SAMI approach has been extended to handle actuator failures.11

REDUNDANT CONTROL AND ACTUATION FAILURE PROBLEM
In high performance dynamic systems, the total number of actuators used may be greater than the number of states to be closely controlled or tracked. This control redundancy generally exists to achieve optimality with respect to control effort. Some modern high performance aircraft have thrust vectoring in addition to the aerodynamic control surfaces. Thrust vectoring is mainly used for vertical take off and landing, but these controls can be used to augment the aerodynamic controls in flight.

In this case of redundant actuation it is still possible to closely track the desired states even if some of the actuators fail, as long as the number

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of active actuators is greater than or equal to the number of states to be tracked. Thus, if it is possible to reconfigure the control after failure, the stability and performance of the system can in theory be maintained.

Strategies to Handle Actuation Failure

Most actuator failure schemes employ some form of failure detection algorithm to detect the failure. A new control effectiveness matrix is estimated and the controller is redesigned by recalculating the control gains.\(^\text{12}\)

This approach depends strongly on the efficacy of the failure detection algorithm. The algorithm may fail to detect a failure, or may give false warning when there is no actuation failure. In contrast, the SAMI controller is constantly updating its parameters so it does not need to specifically detect the failure. The failure is implicitly identified as a change in the parameters of the control effectiveness matrix, and the adaptation mechanism adapts to this change. Therefore, SAMI is a good candidate to address the problem of actuator failure.

Mathematical Modelling of Actuator Freezes

The actuator failures commonly encountered in aircraft and re-entry vehicles such as the X-38 are called control freezes, in which the control surface freezes or remains fixed at a position that may or may not be zero. In such a case, the remaining active actuators must not only compensate for the lack of the desired control effort of the failed actuator, but also cancel the undesired control effect produced if the actuator freezes at any position other than zero.

Control freezes can be modelled by the following mathematical model

\[
\begin{equation}
\mathbf{u}_{\text{applied}} = D\mathbf{u}_{\text{calculated}} + \mathbf{E}
\end{equation}
\]

where \(D \in \mathbb{R}^{m \times m}\) is a constant matrix and \(\mathbf{E} \in \mathbb{R}^m\) is a constant vector for a particular control configuration, but they change and settle to other constant values if a control freeze failure occurs. Here \(m\) is the number of controls. If one wants to model only control freezes then \(D\) should be strictly diagonal as the cross coupling between the calculated value of one control and the applied value of the other control is zero. Since \(\mathbf{u}_{\text{applied}} = \mathbf{u}_{\text{calculated}}\) in the absence of failure, the \(D\) is initialized an identity matrix, and the \(\mathbf{E}\) is a null vector.

\[
\begin{bmatrix}
\mathbf{u}_{a1} \\
\mathbf{u}_{a2} \\
\mathbf{u}_{a3}
\end{bmatrix}
= 
\begin{bmatrix}
D_{11} & 0 & 0 \\
0 & D_{22} & 0 \\
0 & 0 & D_{33}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_{c1} \\
\mathbf{u}_{c2} \\
\mathbf{u}_{c3}
\end{bmatrix}
+ 
\begin{bmatrix}
\mathbf{E}_1 \\
\mathbf{E}_2 \\
\mathbf{E}_3
\end{bmatrix}
\tag{2}
\]

If an actuator freezes, the corresponding diagonal term in the \(D\) matrix should go to zero, and the corresponding element in the vector \(\mathbf{E}\) should go to the constant value at which the control surface has frozen.

This mathematical model can also model damaged control surfaces. Consider a case in which a control surface such as an elevator is damaged by gunfire or some other cause, such that only half of the surface is left. Since the control effectiveness is reduced by half, the corresponding diagonal term in the \(D\) matrix should go to \(0.5\) and the corresponding element in the vector \(\mathbf{E}\) should remain at zero.

By adding this model to the SAMI formulation, a framework can be created to accommodate actuator failures and damage as changes in the parameters of the system.

**CONTROL SATURATION LIMITS**

Adaptive control usually assumes full authority control, and lacks an adequate theoretical treatment for control in the presence of actuator saturation limits. Saturation becomes more critical for adaptive systems than non adaptive systems, since the adaptation is based on the tracking error. Assuming that the dynamics are modelled perfectly and only parametric uncertainties exist in the system, the tracking error has contributions due to the initial error conditions, parametric uncertainties, and saturation. The adaptation scheme adapts only the uncertain parameters, so the error driving the adaptation scheme should not include the error due to saturation. Including the error component due to saturation will cause incorrect adaptation.

To get around this problem two approaches are usually followed:

- Reduce the adaptation rate in the presence of saturation, or completely stop adaptation when the control is saturated.\(^\text{13}\) This stops the incorrect adaptation, but adaptation may be critical when the input is saturated. Consider a case where the parametric uncertainty is high and the control saturates because of these uncertainties. The system may diverge unless the uncertainties are corrected and may not recover from saturation at all.
• If the input is saturated due to an aggressive reference trajectory, the reference command is adjusted so that the input does not saturate.

This paper presents an adaptive control methodology which follows the second school of thought. After separating the system into structured kinematic and dynamic parts, it is noticed that the control directly affects the acceleration. So, the difference between the calculated and the applied control effort due to saturation results in a lack of acceleration produced in the plant as compared to the demanded reference acceleration. This is called the hedge signal. If the hedge signal is removed from the reference, the resulting modified reference can be tracked within saturation limits. The tracking error seen will be only due to the initial error and the parametric uncertainty, hence the controller will adapt correctly.

The SAMI formulation will always remain unsaturated, but the focus now shifts to the stability of the reference model as the reference model now gets dynamically coupled with the plant and the adaptive law. The hedge signal now acts as a disturbance input to the reference model. But, the reference model is a hypothetical mathematical model selected by the designer and is free from the input rate and position saturation constraints. So ensuring stability in the presence of bounded disturbances is simplified.

MATHEMATICAL FORMULATION

This section mathematically formulates a fault tolerant SAMI controller that can handle actuator saturation.

Definition of the Plant and the Reference Model

Consider the mathematical model of the system as follows

\[ M(q, \dot{q}) \ddot{q} = G(q, \dot{q}) + Cu_{app} \]  \hspace{1cm} (3)

\[ \ddot{q} = M^{-1}G + M^{-1}Cu_{app} \]  \hspace{1cm} (4)

where

- \( q \in \mathbb{R}^n = \) vector of generalized coordinates
- \( M(q, \dot{q}) \in \mathbb{R}^{n \times n} = \) mass matrix
- \( G(q, \dot{q}) \in \mathbb{R}^n = \) vector of nonlinear functions of the states used to describe the unforced dynamic behavior
- \( C \in \mathbb{R}^{n \times m} = \) control influence matrix
- \( u \in \mathbb{R}^m = \) vector of the control inputs. \( (m \) should be at least equal to \( n)\).

The dynamics of the system are assumed to be modeled accurately, and only structured parametric uncertainties exist in the model. This second-order differential equation can be split up into kinematic and dynamic parts as

\[ \dot{\sigma} = J(\sigma)\omega \]  \hspace{1cm} (5)

\[ \dot{\omega} = A(\sigma, \omega) + B(\sigma, \omega)u_{app} \]  \hspace{1cm} (6)

where

- \( \sigma \in \mathbb{R}^n = q = \) vector of position level coordinates
- \( \omega \in \mathbb{R}^n = \dot{q} = \) vector of velocity level coordinates
- \( J \in \mathbb{R}^{n \times n} = \) nonlinear transformation relating \( \dot{\sigma} \) and \( \omega \)
- \( A(\sigma, \omega) = M^{-1}G \)
- \( B(\sigma, \omega) = M^{-1}C \)

Note that the most convenient velocity coordinates may not always be the derivatives of the position. Consider the case of an aircraft. The position coordinates are the translational displacements along the earth fixed inertial axis. The velocity level equation is written in the body axis as the moment of inertia remains constant in the body axis at every instant, but the moment of inertia in the inertial axis changes, depending on the orientation of the body. Thus the velocity coordinates are the body-axis velocities. So the velocity coordinate and the position coordinate are related via a series of coordinate axis rotations, hence the term \( J \) in Equation 5.

Substituting for \( u_{app} \) from Equation 1

\[ \dot{\omega} = A(\sigma, \omega) + B(\sigma, \omega)Du_{cal} + B(\sigma, \omega)E \]  \hspace{1cm} (7)

Consider a reference model which is the linearized version of nonlinear reference model at the operating condition.

\[ \begin{bmatrix} \dot{\sigma}_r \\ \dot{\omega}_r \end{bmatrix} = \begin{bmatrix} 0 & J_r \\ A_{r1} & A_{r2} \end{bmatrix} \begin{bmatrix} \sigma_r \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u \]  \hspace{1cm} (8)

Calculating the Control by Dynamic Inversion of the Dynamic Level Equation

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Let the error in the position and the velocity level states be \( s \) and \( x \) respectively

\[
\begin{align*}
    s &= \sigma - \sigma_r, \quad (9) \\
    x &= \omega - \omega_r. \quad (10)
\end{align*}
\]

The control shows up only in the velocity level equation, and hence the corresponding velocity level error. The control affects the position through the integration of Equation 6 via the coupling seen in Equation 5.

\[
\begin{align*}
    \dot{x} &= \dot{\omega} - \dot{\omega}_r \quad (11) \\
    \dot{x} &= A + BDu_{\text{cal}} + BE - \dot{\omega}_r. \quad (12)
\end{align*}
\]

However, the error between the reference and the plant has to go to zero. Hence the dynamics prescribed for \( x \) are

\[
\dot{x} = A_h x + \phi \quad (13)
\]

where

\[
A_h = \text{Hurwitz matrix}, \text{ i.e. } \text{all eigenvalues lie in the open left half plane so that the velocity error dynamics are stable}. \quad (14)
\]

\[
\phi = \text{forcing function on the velocity error dynamics, which helps in achieving the tracking objective. This is discussed in detail later.}
\]

Adding and subtracting \( A_h x + \phi \) on the right hand side

\[
\dot{x} = A_h x + \phi + A + BDu_{\text{cal}} + BE - (\dot{\omega}_r + A_h x + \phi) \quad (15)
\]

Since the quantity in the brackets is known, let

\[
\psi = \dot{\omega}_r + A_h x + \phi \quad (16)
\]

Therefore

\[
\dot{x} = A_h x + \phi + A + BDu_{\text{cal}} + BE - \psi \quad (17)
\]

Using dynamic inversion to solve for the control,

\[
u_{\text{cal}} = (BD)^{-1}(\psi - A - BE) \quad (18)
\]

\[
\begin{itemize}
    \item If the system is under-actuated, \( m \) is less than \( n \), \( (BD) \) will be rank deficient and its inverse cannot be computed.
    \item If the system is over-actuated system (redundant actuation) \( m \) is greater than \( n \), \( (BD) \) will give a minimum norm solution.
\end{itemize}

If there is actuation failure \( (BD) \) will lose rank, as the diagonal element corresponding to the failed control goes to zero. Rank \( (BD) \) is equal to the number of active controls, hence number of active controls after failure should be greater than or at least equal to \( n \).

\[
u_{\text{cal}} = \text{pinv}(BD)(\psi - A - BE) \quad (19)
\]

For redundant actuation, the pseudo inverse is defined as

\[
\text{pinv}(BD) \triangleq (BD)^T \times (BD \times (BD)^T)^{-1} \quad (20)
\]

Definition of Adaptive Learning Parameters

Equation 18 requires the exact values of \( A \) and \( B \) so that the plant dynamics are cancelled exactly. However, the system parameters \( A \) and \( B \) are not known accurately, hence best guesses for \( A \) and \( B \) \( (A_{\text{est}} \text{ and } B_{\text{est}}) \) will be used. Let

\[
u_{\text{cal}} = \text{pinv}(B_{\text{est}}D)(\psi - C_a A_{\text{est}} - B_{\text{est}}E) \quad (21)
\]

Where \( C_a \) is the adaptive learning matrix which will be updated online so as to ensure stability and performance of the system. There exists matrix \( C_a^* \) such that

\[
C_a^* \times A_{\text{est}} = A \quad (22)
\]

The \( D \) matrix that had been introduced to take care of the control freezes can also account for the parametric uncertainty in \( B \) just as the uncertainty in \( A \) is accommodated using \( C_a \). Similarly

\[
B_{\text{est}} \times D^* = B \quad (23)
\]

Actuator Saturation and Modified Reference due to Control Hedging

The control has position limits. So we demand a control which can practically be implemented after saturation.

\[
u_{\text{cal sat}} = \begin{cases} 
    u_{\text{cal}} & \text{if } |u_{\text{cal}}| \leq u_{\text{max}} \\
    u_{\text{max}} \text{sign}(u_{\text{cal}}) & \text{if } |u_{\text{cal}}| > u_{\text{max}} 
\end{cases} \quad (24)
\]
Assuming that $u_{\text{min}} = -u_{\text{max}}$ where $u_{\text{max}}$ is the absolute value of the maximum/minimum control that can be applied. Now, let $\delta$ be the difference between the calculated control and the applied control

$$\delta = u_{\text{cal}} - u_{\text{calcsat}}$$

From Equation 20 and Equation 25,

$$u_{\text{calcsat}} = \text{pinv}(B_{\text{est}}D)(\psi - C_aA_{\text{est}} - B_{\text{est}}E) - \delta$$

Hence

$$\psi = C_aA_{\text{est}} + B_{\text{est}}Du_{\text{calcsat}} + B_{\text{est}}D\delta + B_{\text{est}}E$$

If $u_{\text{calcsat}}$ is the actual control that is demanded, Equation 16 becomes

$$\dot{x} = A_hx + \phi + A + BDu_{\text{calcsat}} + BE - \psi$$

Substituting for $\psi$ in Equation 28 and using Equations 21-23 results in

$$\dot{x} = A_hx + \phi + C_a^*A_{\text{est}} + B_{\text{est}}D^*u_{\text{calcsat}} + B_{\text{est}}E^*$$

$$-C_aA_{\text{est}} - B_{\text{est}}Du_{\text{calcsat}} - B_{\text{est}}D\delta - B_{\text{est}}E$$

Defining

$$\tilde{C}_a \triangleq C_a^* - C_a$$

$$D \triangleq D^* - D$$

$$\tilde{E} \triangleq E^* - E$$

Equation 29 now becomes

$$\dot{x} = A_hx + \phi + \tilde{C}_aA_{\text{est}} + B_{\text{est}}\tilde{D}u_{\text{calcsat}}$$

$$+ B_{\text{est}}E - B_{\text{est}}D\delta$$

(33)

The hedge signal now acts as a disturbance input to the reference model and causes the reference model trajectory to deviate from the desired dynamics. Tracking the desired dynamics throughout the duration of the trajectory is impossible because of the control saturation limits. Demanding the system to track a desired trajectory for which the control saturates for the entire time duration is impractical. Hence for any feasible desired trajectory the control should be unsaturated for some period of time and the hedge signal should be zero. Whenever the control is not in saturation and the hedge is zero, the original desired dynamics are recovered and asymptotic stabilization of the total tracking error $y$ (proved later in the section on Stability Analysis) ensures that the asymptotic tracking for the reference dynamics can be achieved after the control is out of saturation. So finally there is an original desired reference trajectory and a linear controller that makes the modified reference trajectory track the desired reference trajectory. This modified reference now acts as a reference for the nonlinear SAMI controller.

The modified acceleration tracking error is

$$\dot{x} = A_hx + \phi + \tilde{C}_aA_{\text{est}} + B_{\text{est}}\tilde{D}u_{\text{calcsat}} + B_{\text{est}}\tilde{E}$$

(37)

After hedging when the reference trajectory is modified and the control is calculated. The new calculated control $u_{\text{cal}} = u_{\text{calcsat}}$.

Incorporating position into the tracking error

The control is derived from the dynamic part, and the adaptive mechanism will ideally provide perfect velocity tracking. This does not ensure that the position reference will be tracked correctly. The initial errors in position or errors in the velocity, during the transient stage, before perfect velocity tracking is achieved, will cause the position to stray from the reference and no attempts at correcting this error will be made unless this deviation is identified as an error. So, the tracking error should have a contribution from the position level state also. Let the total tracking error be defined as

$$y = \dot{s} + \lambda s$$

(38)

where $\lambda \in \mathbb{R}^{n \times n}$ is a constant positive definite matrix. As $t \to \infty$, if $y$ is driven to zero, it is ensured that $s \to 0$ and $\dot{s} \to 0$.

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Differentiating Equation 9 with respect to time
\[ \dot{s} = \dot{\sigma} - \dot{\sigma}_r \]  
\[ \dot{s} = J\omega - J_r\omega_r \]  
\[ y = J(\omega - \omega_r) + J\omega_r - J_r\omega_r + \lambda \dot{s} \]  

Adding and subtracting \( J\omega_r \) from the right hand side
\[ y = J\omega - J\omega_r - J_r\omega_r + \lambda \dot{s} \]  

By definition \( x = \omega - \omega_r \)
\[ y = Jx + J\omega_r - J_r\omega_r + \lambda \dot{s} \]  

Differentiating Equation 43 to obtain the derivative of the tracking error
\[ \dot{y} = J\dot{x} + J\dot{x} + (\dot{J} - J_r)\omega_r + (J - J_r)\dot{\omega}_r + \lambda \ddot{s} \]  

Substituting for \( \dot{x} \) from Equation 37
\[ \dot{y} = \dot{J}(A_hx + \phi + \tilde{C}_aA_{est} + B_{est}\tilde{D}u_{cal} + B_{est}\tilde{E}) + J\dot{x} + (\dot{J} - J_r)\omega_r + (J - J_r)\dot{\omega}_r + \lambda \ddot{s} \]  

The quantities \( \tilde{C}_a, \tilde{D} \) and \( \tilde{E} \) are not known, while all the other quantities are known. For the tracking error to stabilize, the following dynamics are prescribed to the known quantities. The uncertain quantities will be taken care of with a Lyapunov analysis done later. Thus
\[ JA_hx + J\phi + J\dot{x} + (\dot{J} - J_r)\omega_r + (J - J_r)\dot{\omega}_r + \lambda \ddot{s} = A_hy \]  

Solving for the forcing function \( \phi \):
\[ \phi = J^{-1}(A_hy - \lambda \dot{s} - \dot{J}\omega + J_r\dot{\omega}_r + J_r\omega_r) \]  

Finally, substituting the value of \( \phi \) in Equation 45
\[ \dot{y} = A_hy + J(\tilde{C}_aA_{est} + B_{est}\tilde{D}u_{cal} + B_{est}\tilde{E}) \]  

Taking the derivative of the Lyapunov function
\[ \dot{V} = y^TP\dot{y} + \dot{\tilde{y}}^TP\dot{y} + 2Tr(\tilde{C}_a^TW_1\tilde{C}_a + \tilde{D}^TW_2\tilde{D}) \]  
\[ + 2\tilde{E}^TW_3\ddot{\tilde{E}} \]  

\[ \dot{V} = y^TPA_hy + y^TA_h^TPy + \]  
\[ 2Tr(A_{est}^T\tilde{C}_a^TW_1\dot{\tilde{C}}_a + \tilde{D}^TW_2\dot{\tilde{D}}) \]  
\[ + 2\tilde{E}^TW_3\ddot{\tilde{E}} \]  

\( P \) is selected such that \( PA_h + A_h^TP = -Q \), where \( Q \) is a positive definite matrix. Existence of \( P \) and \( Q \) to satisfy the above relation is guaranteed as \( A_h \) is Hurwitz. Also using the identity:
If \( A \) and \( B \) are row and column matrices respectively, then \( AB = Tr(AB) \):
\[ \dot{V} = -y^TQy + 2Tr(\tilde{C}_a^TW_1\dot{\tilde{C}}_a + \tilde{D}^TW_2\dot{\tilde{D}}) \]  
\[ + 2\tilde{E}^TW_3\ddot{\tilde{E}} \]  

\( \tilde{C}_a = C_a - C_a^* \) cannot be calculated because the values of the actual parameters, such as \( C_a^* \), are not known. So the coefficient of \( C_a^* \) must go to zero. Retaining only the negative definite part \(-y^TQy\) and setting all other terms to zero,
\[ \dot{\tilde{C}}_a = -W_1^{-1}(J^TPyA_{est}^T) \]  
\[ \dot{C}_a^* = -W_1^{-1}(J^TPyA_{est}^T) \]  

However, \( C_a^* \) is assumed to be constant, so that
\[ \dot{\tilde{C}}_a = W_1^{-1}(J^TPyA_{est}^T) \]  

Similarly,
\[ \dot{\tilde{D}} = W_2^{-1}(B_{est}^TW^TPyA_{est}^T) \]  
\[ \dot{\tilde{E}} = W_3^{-1}(B_{est}^TW^TPy) \]  

These are the update equations for the various adaptive learning parameters.

**Stability Analysis**
\[ V = V(y, \tilde{C}_a, \tilde{D}, \tilde{E}), \]  
\[ V = 0 \]  
\[ \tilde{C}_a = 0, \tilde{D} = 0, \text{ and } \tilde{E} = 0. \]  
\( (\text{where 0 is a null vector or null matrix of appropriate dimensions.}) \) But the derivative \( \dot{V} = \dot{V}(y) \) only. \( \dot{V} = 0 \), when \( y = 0 \), irrespective of the values of \( \tilde{C}_a, \tilde{D} \) and \( \tilde{E} \). Hence \( V \) is negative semidefinite.
Thus the adaptive control law (Equation 20) along with the update laws (Equations 55, 56 and 57) ensure global stability. From the properties of $V$ and $\dot{V}$ stated above, we conclude that $y \in L_2 \cap L_\infty$ and $C_a, D$ and $E \in L_\infty$.

$y$ is defined as $y = \dot{s} + \lambda s$. If $(\dot{s} + \lambda s) \in L_2 \cap L_\infty$ then $s \in L_2 \cap L_\infty$, and $\dot{s} \in L_2 \cap L_\infty$. Since the reference trajectories are bounded $(\sigma_r, \dot{\sigma}_r) \in L_\infty$. So from the definition of $s$ and $\dot{s}$, it can be concluded that $(\sigma, \dot{\sigma}) \in L_\infty$. Since $\dot{\sigma} = J_\omega$, $\omega \in L_\infty$, which implies $A(\sigma, \omega)$ and $\phi \in L_\infty$. Equation 48 shows that all the signals in the $\dot{y}$ are bounded hence $\dot{y} \in L_\infty$.

Thus

- $y \in L_2 \cap L_\infty$, and
- $\dot{y} \in L_\infty$.

From Barbalat’s lemma\textsuperscript{17} we conclude that $y \to 0$ as $t \to \infty$. Thus $s \to 0$ and $\dot{s} \to 0$ which $\Rightarrow \sigma \to \sigma_r$ and $\omega \to \omega_r$. Thus the states of the plant converge to the reference and perfect tracking can be achieved.

The system will follow a trajectory, so that the tracking error goes to zero as time goes to infinity. However, $\dot{V}$ is a function of the tracking error only, so when the tracking error becomes zero, the system parameters do not update. The closed-loop system is shown to be globally asymptotically stable for trajectories without singularities. However, the adaptively estimated parameters may not converge to the actual parameters of the system during the duration of the maneuver. Since it is assumed that parameters like $C_a^*, D^*$ and $E^*$ are constants, this formulation works only when the plant parameters are constant with respect to time or slowly time varying, as compared to the rate of update of the adaptive parameters.

**A NONLINEAR SIX-DEGREE-OF-FREEDOM SIMULATION OF AN F-16 TYPE AIRCRAFT**

This section simulates a simultaneous actuator failure and actuator saturation for an F-16 type aircraft.

Definition of Variables and Development of a Nonlinear Mathematical Model of the F-16 type Aircraft

The general equations of motion of an aircraft are derived for the body axis fixed to the airplane, with the origin at the center of gravity.\textsuperscript{18} The orientation and position of the airplane is defined in terms of an inertial reference frame fixed to the earth (Figure 1). Let $dx, dy$ and $dz$ denote the position of the aircraft along the $X, Y$ and $Z$ axes respectively. The angular orientation of the aircraft can be described by a 3-2-1 rotation sequence through the Euler angles $\psi, \theta$ and $\phi$, respectively. The angle-of-attack and the side-slip angle can be defined in terms of the velocity components as (Figure 2).

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right)$$  \hspace{1cm} (58)
$$\beta = \sin^{-1}\left(\frac{v}{V_{res}}\right)$$  \hspace{1cm} (59)

where $V_{res}$ is the resultant velocity. The variables that completely define the state-space for this aircraft model are the position level vector:

$$\sigma = [\phi \ \theta \ \psi \ dx \ dy \ dz]^T$$  \hspace{1cm} (60)

and the velocity level vector:

$$\omega = [p \ q \ r \ u \ v \ w]^T$$  \hspace{1cm} (61)

Following the earlier discussion, it is clear that the number of controls must be greater than the number of velocity level states. So there are eight controls on the hypothetical F-16 type aircraft: right horizontal tail, left horizontal tail, right aileron, left aileron, rudder, and additional three controls for
Fig. 2 Definition of Angle of Attack, $\alpha$ and Side-Slip, $\beta$

thrust vectoring: thrust along the $X$, $Y$, and the $Z$ axes. With a total of eight controls, the control algorithm can tolerate a maximum of two control failures, both of which may fail at arbitrary times.

Defining $S_{\theta} = \sin(\theta)$, $C_{\theta} = \cos(\theta)$, etc. The structured model with separate kinematic and dynamic parts can be written as

1. Kinematic Part:

$$
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}
= \begin{bmatrix}
J_{ang} & 0_{3\times3} \\
0_{3\times3} & J_{lin}
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r \\
u \\
v \\
w
\end{bmatrix}
$$

(62)

Where

$$
J_{ang} =
\begin{bmatrix}
C_{\theta}C_{\psi} & S_{\theta}S_{\phi}C_{\psi} - C_{\phi}S_{\psi} & C_{\phi}S_{\theta}C_{\psi} - S_{\phi}S_{\psi} \\
C_{\theta}S_{\psi} & S_{\theta}S_{\phi}S_{\psi} - C_{\phi}C_{\psi} & C_{\phi}S_{\theta}S_{\psi} + S_{\phi}C_{\psi} \\
-S_{\theta} & S_{\phi}C_{\theta} & C_{\phi}C_{\theta}
\end{bmatrix}
$$

(63)

$$
J_{lin} =
\begin{bmatrix}
1 & S_{\phi}\tan(\theta) & C_{\phi}\tan(\theta) \\
0 & C_{\phi} & -S_{\phi} \\
0 & S_{\phi}\sec(\theta) & C_{\phi}\sec(\theta)
\end{bmatrix}
$$

(64)

$0_{3\times3}$ is a matrix of order $3 \times 3$ with all elements being zero. Equation 62 is in the form of the model, Equation 5

$$
\dot{\sigma} = J(\sigma)\omega
$$

2. Dynamic Part: Let, angular velocity $\omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ and linear velocity $V = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$, with $I$ the moment of inertia of the aircraft about the body axes and $m$ the mass.

The inertia matrix is

$$
I = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix}
$$

(65)

From Euler's rigid body equations,

$$
\dot{\omega} = I^{-1} \begin{bmatrix} L \\ M \\ N \end{bmatrix} - \tilde{\omega}I\omega
$$

(66)

$$
\dot{V} = \frac{1}{m} \begin{bmatrix} X - mgS_{\theta} \\ Y + mgC_{\phi}S_{\psi} \\ Z + mgC_{\phi}C_{\psi} \end{bmatrix} - \tilde{\omega}V
$$

(67)

where $\tilde{\omega}V$ is the matrix representation of the cross-product between vector $\omega$ and vector $V$.

$$
\tilde{\omega} = \begin{bmatrix}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}
$$

(68)

The external moments and forces have contributions due to the aircraft wing and the fuselage that are dependent on the angle of attack, the sideslip angle and the controls.

Consider the total external pitching moment acting on the aircraft

$$
M = \begin{bmatrix}
\frac{1}{2}\rho V^2 S(C_{mm_\theta}\beta + C_{mm_p}p + C_{mm_r}r) \\
part1(unforced) \\
\end{bmatrix}
+ \begin{bmatrix}
C_{mm_{cont}}u \\
part2
\end{bmatrix}
$$

(69)

where $\frac{1}{2}\rho V^2 S$ is the product of dynamic pressure and the wing area. $C_{mm_\theta}, C_{mm_p}, C_{mm_r}$ are the corresponding stability coefficients. $C_{mm_{cont}}$ is a row vector. The $i^{th}$ element of this vector represents the contribution to the
pitching moment due to unit deflection of the $i^{th}$ control.

Since external moment and force contributions in part one of Equation 69 play a role in the unforced dynamic behavior they form a part of the A matrix, while the external force and moment contributions in part two of Equation 69 form a part of the B matrix.

The dynamic part of the equation of motion can now be written as

$$
\begin{bmatrix}
\dot{\omega} \\
\dot{V}
\end{bmatrix}
= 
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
+ 
\begin{bmatrix}
C_{u_{cont}} \\
C_{mm_{cont}} \\
C_{nn_{cont}} \\
C_{x_{cont}} \\
C_{y_{cont}} \\
C_{z_{cont}}
\end{bmatrix}
\begin{bmatrix}
u
\end{bmatrix}
$$

Where

$$A_1 = I^{-1}
\begin{bmatrix}
L \\
M
\end{bmatrix}
\new_{unforced}
- \ddot{\omega}J\omega
\tag{70}$$

$$A_2 = \frac{1}{m}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\new_{unforced}
+ 
\begin{bmatrix}
-mgS_\theta \\
mgC_{\theta}S_\phi \\
mgC_{\phi}
\end{bmatrix}
- \ddot{\omega}V
\tag{71}$$

which has the same form as Equation 6

$$\ddot{\omega} = \Lambda(\sigma, \omega) + B(\sigma, \omega)u_{app}$$

as desired.

Numerical Example: Thrust Vectoring Generic F-16

The example consists of an aileron actuator failure which occurs from an initially trimmed flight condition. The aircraft model is a generic F-16 which has been modified by augmenting the existing flight controls with three additional controls: The thrust vectoring components along the X, Y and the Z axis. The flight condition is Mach 0.8 at 50,000 ft. The simulation is done with parametric errors in the vector A and the B matrix, initial error conditions, and the right aileron fails at time equal to five seconds and settles down at a constant value of negative one degree. Due to this failure the aircraft starts rolling rapidly. In an attempt to recover from this failure some of the remaining controls saturate. Now, if the adaptation is turned off during saturation, the system diverges. But by doing control hedging, it is possible to continue adapting and recover from the failure, in spite of the saturation.

Fig. 3 Time Histories of the Angular States of the Aircraft in Steady Level Flight with Simultaneous Actuator Failure and Actuator Saturation (Case 1)

Fig. 4 Time Histories of the Linear States of the Aircraft in Steady Level Flight with Simultaneous Actuator Failure and Actuator Saturation (Case 1)

The first set of graphs for Case 1 (Figures 3-5) show the time histories for the case where adaptation is stopped on saturation. From Figure 5 it is seen that the controls saturate at time approximately equal to 6.5 seconds. All the angular and the linear states start diverging at this time as seen from Figures 3 and 4 respectively. This is because the controls saturated due to the failure, and if adaptation is stopped after failure, the system cannot recover.

The Figures 6-8 for Case 2 show the time histories of the states and the controls when the reference is modified by the hedging signal. The basic idea
here is to modify an aggressive reference trajectory so that the resulting trajectory can be tracked within control limits. When the right aileron has failed and the aircraft is rolling rapidly large control deflections are needed to bring it back to horizontal position immediately. From Figure 6 it is seen that the hedging signal modifies the reference trajectory in such a way that the trajectory becomes less aggressive. Instead of returning to the horizontal position immediately, the reference trajectory demands that the aircraft return to the horizontal position smoothly. This is seen from the trajectory for the Euler angle $\phi$ from Figure 6. Figure 7 shows that the linear states of the aircraft converge to the reference and Figure 8 shows that the controls are within bounds.

**CONCLUSIONS**

This paper derived and validated a Structured Adaptive Model Inversion Control Law that simultaneously handles actuator failure and actuator saturation. Fault Tolerance is achieved by adding a mathematical model of controller failure to the plant model, so that the failure can be identified as a change in the system parameters. Correct adaptation in presence of saturation is achieved by modifying the reference so that saturation is avoided and the tracking error due to saturation does not influence the update of the learning parameters. The control law ensures global asymptotic stability of the tracking errors. Even though, the adaptive learning parameters may not converge to the actual parameters within the duration of the maneuver.

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Fig. 8 Time Histories of the Controls of the Aircraft in Steady Level Flight with Simultaneous Actuator Failure and Actuator Saturation (Case 2)

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REFERENCES


