SELECTION OF DESIRED DYNAMICS FOR DYNAMIC INVERSION CONTROLLED RE-ENTRY VEHICLES

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Aerospace Engineering

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Overview

- PROJECT BACKGROUND
- DYNAMIC INVERSION
- DESIRED DYNAMICS
  - Proportional
  - Proportional Plus Integral
  - Flying Qualities
  - Ride Qualities
- DESIGN EXAMPLE
  - Time Domain
  - Frequency Domain
  - Fragility Analysis
- CONCLUSIONS
- FURTHER WORK
Project Background (1)

- **X-38 CREW RETURN VEHICLE**
  - Serve As a Lifeboat for the International Space Station

- **CURRENT FLIGHT SOFTWARE**
  - Multi Variable Controller (MACH)

- **CHARACTERISTICS**
  - Lifting Body Configuration
  - Autonomous Control
  - Large Flight Envelope
  - Large Uncertainties

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Docked with Space Station

De-Orbit

Re-Entry

Maneuvering

Landing

General Re-Entry Profile
Project Background (2)

• PROGRAM
  ➢ “Synthesis and Evaluation of Robust Dynamic Inversion Flight Controllers for X-38 Class Re-Entry Vehicles”
  ➢ NASA Johnson GN&C Design & Analysis Branch
    — Mark Hammerschmidt

• DESIRED DYNAMICS RESEARCH OBJECTIVES
  ➢ Investigate the effects and use of different forms of desired dynamics
  ➢ Develop methodology, guidelines, and recommendations
  ➢ Include in Dynamic Inversion tutorial document
    — self-contained with necessary theory, implementation detail, coding examples, and generic system and re-entry vehicle design examples
    — converted to NASA TP
Dynamic Inversion

CONCEPT

• CONSIDER SYSTEMS THAT ARE AFFINE IN CONTROL
  \[ \dot{x} = f(x) + g(x)u \]

• SOLVE FOR THE CONTROL VECTOR \( u \) EXPLICITLY
  \[ u = g(x)^{-1}\{\dot{x} - f(x)\} \]

• MAKE THE FOLLOWING ASSUMPTIONS:
  – Perfect Measurement of the Feedback States
  – An Invertible Control Matrix
  – Perfect Actuator Dynamics
  – Perfect Modeling of the System

• SPECIFY THE RATE OF THE DESIRED STATES
  \[ u = g^{-1}(x)[\dot{x}_{\text{des}} - f(x)] \]
Procedure

1. Open-Loop
2. Single DI Loop/2-Time Scale
3. Internal Dynamics
4. Desired Dynamics Choices
5. Inner-Loop Design
6. Robust Outer-Loop Design
7. Design Specifications
Desired Dynamics (1)

• THE AUGMENTED DYNAMICS THAT REPLACE THE INHERENT PLANT DYNAMICS OF THE VEHICLE
  - Designer Specified
  - Currently No Guidelines For Selection
  - The Rate of The Desired Control Variable Is Specified, Not The Control Variable Itself

• DI CURRENT APPLICATIONS:

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Desired Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-18 (derivative)</td>
<td>Proportional plus Integral</td>
</tr>
<tr>
<td>ICE (tailless fighter)</td>
<td>Proportional plus Integral</td>
</tr>
<tr>
<td>VAAC Harrier</td>
<td>Proportional</td>
</tr>
<tr>
<td>Prototype Aircraft</td>
<td>Flying Qualities</td>
</tr>
</tbody>
</table>

• Also:
  - F-16
  - Su-27
  - Missiles
Desired Dynamics (2)

- PROPORTIONAL
  - Desired Dynamics Are Proportional To The Error Between The Command and Its Feedback
  \[
  \dot{x}_{\text{des}}(s) = K_{\text{PROP}}(x_{\text{cmd}} - x)
  \]
  
  - Simplest Choice Of Desired Dynamics
  - Closed Loop Transfer Function:
  \[
  \frac{x}{x_{\text{cmd}}}(s) = \frac{K_{\text{PROP}}}{s + K_{\text{PROP}}}
  \]
  - Permits placement of a single pole
  \[
  s = -K_{\text{PROP}}
  \]
**Desired Dynamics (3)**

- **PROPORTIONAL INTEGRAL (PI)**
  - Popular Choice in Fighter Applications
  - Form Adapted From Honeywell Study:

\[
\dot{x}_{\text{des}}(s) = K_{\text{pl}} \left( \frac{1}{2} x_{\text{cmd}} - x \right) + \frac{K_{\text{pl}}^2}{4s} (x_{\text{cmd}} - x)
\]

- Closed-Loop Transfer Function:

\[
\frac{x}{x_{\text{cmd}}}(s) = \frac{1}{2} \frac{K_{\text{pl}}}{s + \frac{1}{2} K_{\text{pl}}}
\]

- Permits placement of a single pole:

\[
s = -\frac{1}{2} K_{\text{pl}}
\]
**Desired Dynamics (4)**

- **FLYING QUALITIES**
  - How The Aircraft Response Feels To The Pilot
    \[ \dot{x}_{\text{des}}(s) = \frac{K_{\text{FQ}}(s + a)}{s^2 + bs + c} \left( x_{\text{cmd}} - x \right) \]
    
  - Closed-Loop Transfer Function:
    \[ \frac{x}{x_{\text{cmd}}}(s) = \frac{K_{\text{FQ}}(s + a)}{s^3 + bs^2 + \left( c + K_{\text{FQ}} \right)s + K_{\text{FQ}}a} \]
    
    - Permits placement of multiple poles and a zero
    - Note: Selecting \( a=0 \) reduces the closed-loop System to 2\(^{\text{nd}}\) Order

- Select Desired Closed-Loop Flying Qualities Based on 2\(^{\text{nd}}\) Order System
- Zero at The Origin Was Not Feasible
  - Selected a new zero location, and adjusted the gain
Desired Dynamics (5)

• **RIDE QUALITIES**
  - Perception Of A Ride Based Upon The Vertical And Lateral Accelerations Experienced By The Passenger
  - Dynamics Take The Form Of A Low Pass Filter
    - Filter out high frequencies (e.g. accelerations)
      \[
      \dot{x}_{des}(s) = \frac{K_{RQ}}{s + b} (x_{cmd} - x)
      \]

- Closed-Loop Transfer Function
  \[
  \frac{x}{x_{cmd}}(s) = \frac{K_{RQ}}{s^2 + bs + K_{RQ}}
  \]
  - Permits placement of 2 poles
    \[
    s_{1,2} = -0.5 \pm 0.5 \sqrt{b^2 - 4K_{RQ}} i
    \]
Desired Dynamics (6)

- RIDE QUALITIES (con’t.)
- CONTROL ANTICIPATION PARAMETER
  - For Highly Augmented Airplanes, CAP Replaces Short Period Requirements
  - Select a CAP Value and Desired $\zeta_{SP}$
  - Calculate the Desired Dynamics Required to Achieve This Cap

\[
\text{CAP} = \frac{\omega^2_{n,SP}}{n_\alpha}
\]

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inherent Dynamics</td>
<td>Desired Dynamics</td>
<td></td>
</tr>
</tbody>
</table>

CAP (1/Sec²) vs. Short Period Damping Ratio, $\zeta_{SP}$
Desired Dynamics (7)

- **MOTION SICKNESS**
  - Passengers Susceptible to Sickness Within a Certain Frequency Range
  - Attenuate Signals Within This Region
    - Attenuate $\sigma$ Response

![Graph showing desired dynamics and frequency response.](image)
Design Example

- **LTI CONTROLLER DESIGN**
- **LONGITUDINAL MODEL**
  - Stable short period, phugoid modes
  - Light short period damping
    \[
    \begin{bmatrix}
    \dot{u} \\
    \dot{\alpha} \\
    \dot{q} \\
    \dot{\theta}
    \end{bmatrix} =
    \begin{bmatrix}
    -0.0335 & -22.5 & 0 & -32.2 \\
    0 & -0.0944 & 1.0 & 0 \\
    0 & -1.94 & -0.188 & 0 \\
    0 & 0 & 1 & 0
    \end{bmatrix}
    \begin{bmatrix}
    u \\
    \alpha \\
    q \\
    \theta
    \end{bmatrix} +
    \begin{bmatrix}
    -8.83 \\
    -0.0196 \\
    -2.02 \\
    0
    \end{bmatrix} \delta_e
    \]
  - **LAT/D MODEL**
    - Unstable Dutch roll mode
    - Stable roll, spiral modes
    \[
    \begin{bmatrix}
    \dot{\beta} \\
    \dot{p} \\
    \dot{r} \\
    \dot{\phi}
    \end{bmatrix} =
    \begin{bmatrix}
    -0.048 & 0 & -1 & 0.032 \\
    -29.35 & -0.15 & 0.082 & 0 \\
    0.11 & 0.012 & -0.091 & 0 \\
    0 & 1 & 0 & 0
    \end{bmatrix}
    \begin{bmatrix}
    \beta \\
    p \\
    r \\
    \phi
    \end{bmatrix} +
    \begin{bmatrix}
    0 & 0.0048 \\
    3.13 & 3.35 \\
    -0.22 & -0.91 \\
    0 & 0
    \end{bmatrix} \delta_a
    \]
- **CONTROL SURFACES**
  - Elevons and rudders
  - Second-order actuator model
    \[
    \zeta = 0.707, \ \omega_n = 26 \text{ rad/sec}
    \]

\[\delta_{R,L} \quad \delta_{R,R} \quad \delta_{E,L,L} \quad \delta_{E,L,R} \quad \delta_{E,R,L} \quad \delta_{E,R,R} \]
Control Surface
Positive Deflections

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Design Specifications

- **TIME DOMAIN:**
  - Class II Vehicle - Phase C
  - Flying Quality Level 1
  - Effector Limits:

<table>
<thead>
<tr>
<th></th>
<th>Position Limits</th>
<th>Rate Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper</td>
<td>Lower</td>
</tr>
<tr>
<td>Bodyflap</td>
<td>0.0°</td>
<td>45.0°</td>
</tr>
<tr>
<td>Rudder</td>
<td>-25.0°</td>
<td>25.0°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Command</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>α to 2°</td>
<td>0.63</td>
</tr>
<tr>
<td>q to 1°/sec</td>
<td>0.63</td>
</tr>
<tr>
<td>φ to 10°</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Design Specifications

- **FREQUENCY DOMAIN**
  - Zero Steady State Error
  - Attenuate Frequencies by 20 dB up to 0.1 Rad/sec
  - Model Accurate Within 10% up to a Frequency of 2 Rad/sec
    - Grows without bound at 20 dB/decade thereafter
    \[ m(\omega) = \frac{s + 2}{20} \]
  - Attenuate Signals In Motion Sickness Range:
    \[ 0.62 \text{ s}^{-1} \leq \omega \leq 6.2 \text{ s}^{-1} \]

![Graph showing singular values and performance robustness requirements](attachment:graph.png)
Results (1)

- TIME DOMAIN

Pitch Rate Controllers

Angle-of-Attack Controllers
Results (2)

- TIME DOMAIN

Bank Angle Controllers
Results (3)

- FREQUENCY DOMAIN

Pitch Rate Controllers

Angle-of-Attack Controllers
Results (4)

- FREQUENCY DOMAIN

Bank Angle Controllers
Results (5)

- **COMFORT INDEX**
  - Based On Previous Ride Qualities Efforts (1975)
    \[ C = 2.1 + 17.1 \bar{a}_y + 17.2 \bar{a}_z \]
  - Assume:
    - Longitudinal controller: \( \bar{a}_y = 0 \)
    - Lateral/Directional controller: \( \bar{a}_z = 0 \)

<table>
<thead>
<tr>
<th>Rating</th>
<th>7-Point Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Uncomfortable</td>
<td>7</td>
</tr>
<tr>
<td>Uncomfortable</td>
<td>6</td>
</tr>
<tr>
<td>Somewhat Uncomfortable</td>
<td>5</td>
</tr>
<tr>
<td>Acceptable (Neutral)</td>
<td>4</td>
</tr>
<tr>
<td>Somewhat Comfortable</td>
<td>3</td>
</tr>
<tr>
<td>Comfortable</td>
<td>2</td>
</tr>
<tr>
<td>Very Comfortable</td>
<td>1</td>
</tr>
</tbody>
</table>
Comfort Index Results

- Pitch Rate Controllers
- Angle-of-Attack Controllers
- Bank Angle Controllers

Uncomfortable
Neutral
Comfortable
Fragility Analysis

- **FRAGILITY** (a.k.a. CONTROLLER SENSITIVITY)
  - Sensitivity to Errors In The Controller
  - Not Robustness Related To Modeling Uncertainty And System Perturbations
  - No literature in regard to DI controllers

- **CAUSAL FACTORS**
  - Discretization
  - Roundoff
  - Wordlength Limits

\[ u = B^{-1}_{\text{model}} \{ \dot{x}_{\text{des}} - A_{\text{model}} x \} \]
Fragility Analysis (2)

- SIMULATION

Errors In Assumed Model

Errors In $g(x)^{-1}$
Conclusions (1)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Satisfied Most Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Rate</td>
<td>Flying Qualities</td>
</tr>
<tr>
<td></td>
<td>Ride Qualities</td>
</tr>
<tr>
<td>Angle-of-Attack</td>
<td>Ride Qualities</td>
</tr>
<tr>
<td></td>
<td>Proportional</td>
</tr>
<tr>
<td>Bank Angle</td>
<td>Proportional</td>
</tr>
</tbody>
</table>

- **FRAGILITY RESULTS**
  - Controllers are more sensitive to errors present within the controller itself, than errors arising from model perturbations or disturbances.
  - Particularly sensitive to errors resulting from the inversion of the control distribution matrix.
Conclusions (2)

<table>
<thead>
<tr>
<th>Desired Dynamics</th>
<th>Satisfactory Across The Board</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional</td>
<td>Control, Step Response</td>
</tr>
<tr>
<td>Proportional Integral</td>
<td>Control</td>
</tr>
<tr>
<td>Flying Qualities</td>
<td>Control, Stability Robustness, Motion Sickness</td>
</tr>
<tr>
<td>Ride Qualities</td>
<td>Control, Stability Robustness, Motion Sickness</td>
</tr>
</tbody>
</table>

• BEST OVERALL CANDIDATES
  - Longitudinal Controller: Ride Quality
  - Lateral/Directional Controller: Proportional
Recommendations

• RIGOROUS TESTING OF “WORST-CASE” SCENARIOS
  ➢ Simultaneously Perturb Plant While Adding Uncertainties in the Controller

• NONLINEAR DESIGN AND EVALUATION
  ➢ Determine Effect Of Desired Dynamics In DI Controllers
  ➢ Nonlinear 6-DOF simulation

• EXTEND CONTROLLER FRAGILITY
  ➢ Evaluate Digitization Effect on DI Controllers.
  ➢ Performance of Discrete Desired Dynamics